MATHEMATICS-X

MODULE-5

INDEX

Shrivastava Classes, D-27, Near JVTS Garden,Chattarpur Extension
New Delhi - 110074
9810934436 , 8076575278 , 8700391727

TRIGONOMETRY

INTRODUCTION

Trigonometry is an important branch of mathematics. It is a combination of three Greek words 'Tri (Three) + gon(sides) + metron (measure)'

Thus, in trigonometry, we deal with the measurements of the sides and the angles of a triangle.

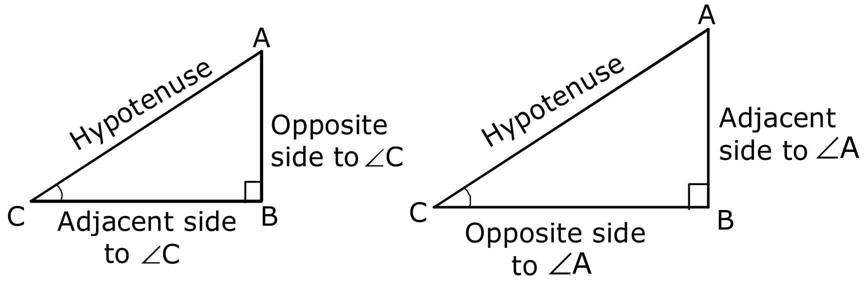
Note :

1. Angle measured in anticlockwise direction is taken as positive angle.
2. Angle measured in clockwise direction is taken as negative angle.

TRIGONOMETRIC RATIOS

We will define certain ratios involving the sides for an acute angle of a right-angled triangle, called trigonometric ratios. Consider a right triangle ABC, right-angled at B.

Now, the side opposite to $\angle B$ i.e., 90° is called the hypotenuse. For $\angle ACB$ or $\angle C$ (an acute angle), AB is the opposite side and CB is the adjacent side. Position of sides change, if we consider $\angle A$ instead of $\angle C$, as shown in the figure i.e., for an acute angle A, CB is the opposite side an AB is the adjacent side. There are six different trigonometric ratios of an angle. These trigonometric ratios are named as sine, cosine, tangent, cosecant, secant and cotangent. For an acute angle C, the six different trigonometric ratios are defined as



sine of $\angle C = \dfrac{\text{opposite side}}{\text{hypotenuse}}$ i.e., $\sin C = \dfrac{AB}{AC}$, cosine of $\angle C = \dfrac{\text{adjacent side}}{\text{hypotenuse}}$ i.e., $\cos C = \dfrac{CB}{AC}$

tangent of $\angle C = \dfrac{\text{opposite side}}{\text{adjacent side}}$ i.e., $\tan C = \dfrac{AB}{CB}$, cosecant of $\angle C = \dfrac{\text{hypotenuse}}{\text{opposite side}}$ i.e., $\text{cosec } C = \dfrac{AC}{AB}$

Secant of $\angle C = \dfrac{\text{hypotenuse}}{\text{adjacent side}}$ i.e., $\sec C = \dfrac{AC}{CB}$, Cotangent of $\angle C = \dfrac{\text{adjacent side}}{\text{opposite side}}$ i.e., $\cot C = \dfrac{CB}{AB}$

Note :

1. Each trigonometric ratio is a real number. It has no unit.
2. cos C is not the product of cosine and C, Only symbol cosine has no meaning

RELATIONS BETWEEN TRIGONOMETRIC RATIOS OF AN ANGLE (θ)

1. Reciprocal relations:

(i) $\text{cosec } \theta = \dfrac{\text{hypotenuse}}{\text{opposite side}} = \dfrac{1}{\dfrac{\text{opposite side}}{\text{hypotenuse}}} = \dfrac{1}{\sin\theta}$ i.e., $\text{cosec } \theta = \dfrac{1}{\sin\theta}$ or $\sin \theta = \dfrac{1}{\text{cosec}\theta}$

Similarly, we have

(ii) $\sec \theta = \dfrac{1}{\cos\theta}$ or $\cos \theta = \dfrac{1}{\sec\theta}$

(iii) $\cot \theta = \dfrac{1}{\tan\theta}$ or $\tan \theta = \dfrac{1}{\cot\theta}$

2. Quotient relations:

(i) $\tan\theta = \dfrac{\text{opposite side}}{\text{adjacent side}}$

$\Rightarrow \tan\theta = \dfrac{\dfrac{\text{opposite side}}{\text{hypotenuse}}}{\dfrac{\text{adjacent side}}{\text{hypotenuse}}}$

$\left[\begin{array}{l}\text{By dividing the numerator and} \\ \text{demo min ator of RHS by hypotenuse}\end{array}\right]$ $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

Similarly, we have (ii) $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Note :

1. $(\sin\theta)^n$ is generaly written as $\sin^n\theta$, n being a positive integer. Similarly, for other trigonometric ratios.
2. The values of the trigonometic ratios of an angle do not vary with the length of the sides of the triangle, if angle remains the same.
3. $\sec\theta = (\cos\theta)^{-1} \neq \cos^{-1}\theta$ [$\cos^{-1}\theta$ is called inverse relation]

TRIGONOMETRIC RATIOS OF SOME SPECIFIC ANGLES:

(i) Trigonometric ratios of 0°

$\sin 0° = 0$, $\cos 0° = 1$, $\tan 0° = 0$, cosec 0° = not defined, $\sec 0° = 1$, cot 0° = not defined

(ii) Trigonometric ratios of 30°

$\sin 30° = \dfrac{1}{2}$, $\cos 30° = \dfrac{\sqrt{3}}{2}$, $\tan 30° = \dfrac{1}{\sqrt{3}}$, $\text{cosec } 30° = 2$, $\sec 30° = \dfrac{2}{\sqrt{3}}$, $\cot 30° = \sqrt{3}$

(iii) Trigonometric ratios of 45°

$\sin 45° = \dfrac{1}{\sqrt{2}}$, $\cos 45° = \dfrac{1}{\sqrt{2}}$, $\tan 45° = 1$, $\text{cosec } 45° = \sqrt{2}$, $\sec 45° = \sqrt{2}$, $\cot 45° = 1$

(iv) Trigonometric ratios of 60°

$\sin 60° = \dfrac{\sqrt{3}}{2}$, $\cos 60° = \dfrac{1}{2}$, $\tan 60° = \sqrt{3}$,

$\text{cosec } 60° = \dfrac{2}{\sqrt{3}}$, $\sec 60° = 2$, $\cot 60° = \dfrac{1}{\sqrt{3}}$

(v) Trigonometric ratios of 90°

$\sin 90° = 1$, $\cos 90° = 0$, tan 90° = not defined, $\text{cosec } 90° = 1$,

sec 90° = not defined, $\cot 90° = 0$

Table of values of trigonometric ratios of 0°, 30°, 45°, 60° and 90°

θ	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\cot\theta$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\text{cosec}\,\theta$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Aid to memory : The following table enable us to memorise the values more easily.

θ	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$	$\sqrt{\frac{0}{4}}$	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{2}{4}}$	$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{4}{4}}$
$\cos\theta$	$\sqrt{\frac{4}{4}}$	$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{2}{4}}$	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{0}{4}}$
$\tan\theta$	$\sqrt{\frac{0}{4-0}}$	$\sqrt{\frac{1}{4-1}}$	$\sqrt{\frac{2}{4-2}}$	$\sqrt{\frac{3}{4-3}}$	not defined

The values of $\text{cosec}\,\theta$, $\sec\theta$, and $\cot\theta$ are not given as these are reciprocals of $\sin\theta$, $\cos\theta$ and $\tan\theta$ respectively.

Note:

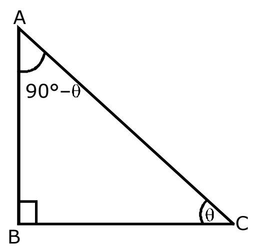
(i) Value of $\sin\theta$ increases from 0 to 1 when θ increases from 0° to 90°.

(ii) Value of $\cos\theta$ decreases from 1 to 0 when θ increases from 0° to 90°

TRIGONOMETRIC RATIOS OF COMPLEMENTARY ANGLES :

In the ΔABC, right-angled at B, if $\angle C = \theta$,

then $\quad \angle A = 180° - (90° + \theta)$



$\Rightarrow \quad \angle A = 180° - 90° - \theta = 90° - \theta,$

Look at the following trigonometrical ratios for θ and $90° - \theta$ and compare.

$\sin\theta = \frac{AB}{AC}$; $\sin(90° - \theta) = \frac{BC}{AC}$; $\cos\theta = \frac{BC}{AC}$; $\cos(90° - \theta) = \frac{AB}{AC}$

$\tan\theta = \frac{AB}{BC}$; $\tan(90° - \theta) = \frac{BC}{AB}$; $\cot\theta = \frac{BC}{AB}$; $\cot(90° - \theta) = \frac{AB}{BC}$

$\sec\theta = \frac{AC}{BC}$; $\sec(90° - \theta) = \frac{AC}{AB}$; $\text{cosec}\,\theta = \frac{AC}{AB}$; $\text{cosec}(90° - \theta) = \frac{AC}{BC}$

Comparing the two tables, we have

$\sin(90° - \theta) = \cos\theta$; $\cot(90° - \theta) = \tan\theta$, $\cos(90° - \theta) = \sin\theta$; $\sec(90° - \theta) = \text{cosec}\,\theta$

$\tan(90° - \theta) = \cot\theta$; $\text{cosec}(90° - \theta) = \sec\theta$

TRIGONOMETRIC IDENTITIES

(Fundamental Identities)

(i) $\sin^2\theta + \cos^2\theta = 1$, (ii) $1 + \tan^2\theta = \sec^2\theta$, (iii) $1 + \cot^2\theta = \text{cosec}^2\theta$

In ΔABC, right-angled at B by pythagoras Theorem

$AB^2 + BC^2 = AC^2$ ...(i)

(i) dividing (i) throughout by AC^2

$$\left(\frac{AB}{AC}\right)^2 + \left(\frac{BC}{AC}\right)^2 = 1 \quad \Rightarrow \sin^2\theta + \cos^2\theta = 1$$

(ii) dividing (i) by BC^2

$$\left(\frac{AB}{BC}\right)^2 + \left(\frac{BC}{BC}\right)^2 = \left(\frac{AC}{BC}\right)^2 \quad \Rightarrow \tan^2\theta + 1 = \sec^2\theta$$

(iii) dividing (i) by AB^2

$$\left(\frac{AB}{AB}\right)^2 + \left(\frac{BC}{AB}\right)^2 = \left(\frac{AC}{AB}\right)^2 \quad \Rightarrow 1 + \cot^2\theta = \text{cosec}^2\theta$$

Note :

(i) $\sin^2\theta + \cos^2\theta = 1$ $\Rightarrow \sin^2\theta = 1 - \cos^2\theta$ or $\cos^2\theta = 1 - \sin^2\theta$

(ii) $\sec^2\theta = 1 + \tan^2\theta$ $\Rightarrow 1 = \sec^2\theta - \tan^2\theta$ or $\tan^2\theta = \sec^2\theta - 1$

(iii) $\text{cosec}^2\theta = 1 + \cot^2\theta$ $\Rightarrow 1 = \text{cosec}^2\theta - \cot^2\theta$ or $\cot^2\theta = \text{cosec}^2\theta - 1$

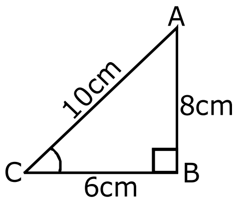
SOLVED PROBLEMS

Ex.1 Using the information given in fig. write the values of all trigonometric ratios of angle C.

Sol. Using the definition of t-ratios,

$\sin C = \frac{AB}{AC} = \frac{8}{10} = \frac{4}{5}$; $\cos C = \frac{BC}{AC} = \frac{6}{10} = \frac{3}{5}$

$\tan C = \frac{AB}{BC} = \frac{8}{6} = \frac{4}{3}$; $\cot C = \frac{BC}{AB} = \frac{6}{8} = \frac{3}{4}$



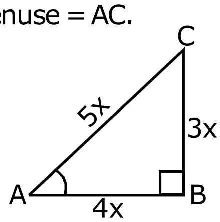
$\sec C = \frac{AC}{BC} = \frac{10}{6} = \frac{5}{3}$ and $\operatorname{cosec} C = \frac{AC}{AB} = \frac{10}{8} = \frac{5}{4}$

Ex.2 In a right ΔABC, if $\angle A$ is acute and $\tan A = \frac{3}{4}$, find the remaining trigonometric ratios of $\angle A$.

Sol. Consider a ΔABC in which $\angle B = 90°$.

For $\angle A$, we have :

Base = AB, perpendicular = BC and Hypotenuse = AC.



$\therefore \quad \tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{3}{4}$

$\Rightarrow \quad \frac{BC}{AB} = \frac{3}{4}$.

Let, BC = 3x units and AB = 4x units.

Then, $AC = \sqrt{AB^2 + BC^2}$

$= \sqrt{(4x)^2 + (3x)^2}$

$= \sqrt{25x^2} = 5x$ units.

$\sin A = \frac{P}{H} = \frac{3}{5}$; $\operatorname{Cosec} A = \frac{H}{P} = \frac{5}{3}$

$\cos A = \frac{B}{H} = \frac{4}{5}$; $\sec A = \frac{H}{B} = \frac{5}{4}$

$\tan A = \frac{P}{B} = \frac{3}{4}$; $\cot A = \frac{B}{P} = \frac{4}{3}$

Ex.3 In a ΔABC, right angled at B, if $\tan A = \frac{1}{\sqrt{3}}$, find the value of

(i) sinA cosC + cosA sinC

(ii) cosA cosC – sinA sinC.

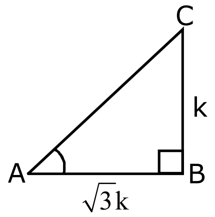
Sol. We know that

$\tan A = \frac{BC}{AB} = \frac{1}{\sqrt{3}}$

$\therefore \quad BC : AB = 1 : \sqrt{3}$

Let BC = k and AB = $\sqrt{3}\,k$

Then, AC = $\sqrt{AB^2 + BC^2}$ ...(Phythagoras theorem)



$= \sqrt{(\sqrt{3}k)^2 + (k)^2} = \sqrt{3k^2 + k^2}$

$= \sqrt{4k^2} = 2k$

Now, $\sin A = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$

$\cos A = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

$\sin C = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$ and $\cos C = \frac{BC}{AC} = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$

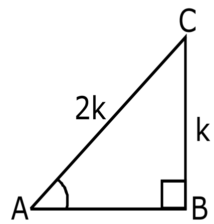
(i) $\sin A \cos C + \cos A \sin C = \frac{1}{2} \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = 1$

(ii) $\cos A \cos C - \sin A \sin C = \frac{\sqrt{3}}{2}\,\frac{1}{2} - \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$

Ex.4 If $\sin A = \frac{1}{2}$, verify that $2 \sin A \cos A = \frac{2\tan A}{1 + \tan^2 A}$

Sol. We know that

$\sin A = \frac{BC}{AC} = \frac{1}{2}$



Let BC = k and AC = 2k

$\therefore$ AB = $\sqrt{AC^2 - AB^2}$ ...(Pythagoras theorem)

$= \sqrt{(2k)^2 - k^2} = \sqrt{4k^2 - k^2} = \sqrt{3k^2} = \sqrt{3}k$

Now $\cos A = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$ and $\tan A = \frac{BC}{AB} = \frac{k}{\sqrt{3}k} = \frac{1}{\sqrt{3}}$

Now $2 \sin A \cos A = 2 . \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$ ...(i)

and $\frac{2\tan A}{1 + \tan^2 A} = \frac{2 \cdot \frac{1}{\sqrt{3}}}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$

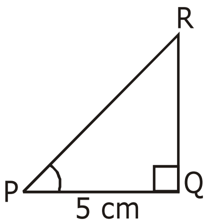
$= \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{\sqrt{3}}{2}$ ...(ii)

Hence from (i) and (ii)

$2 \sin A \cos A = \frac{2\tan A}{1 + \tan^2 A}$.

Ex.5 In ΔPQR, right angled at Q, PR + QR = 25 cm and PQ = 5 cm. Find the value of sinP, cosP and tanP.

Sol. We are given



$PR + QR = 25$ cm

$\therefore$ $PR = (25 - QR)$ cm

By Pythagoras theorem,

$PR^2 = QR^2 + PQ^2$

or $(25 - QR)^2 = QR^2 + 5^2$

or $625 + QR^2 - 50\,QR = QR^2 + 25$

or $50\,QR = 625 - 25 = 600$

$\therefore$ $QR = 12$ cm.

and $PR = (25 - 12)$ cm $= 13$ cm

Now $\sin P = \frac{QR}{PR} = \frac{12}{13}$

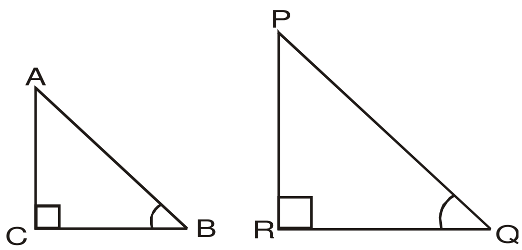
$\cos P = \frac{PQ}{PR} = \frac{5}{13}$

and $\tan P = \frac{QR}{PQ} = \frac{12}{5}$

Ex.6 If $\angle B$ and $\angle Q$ are are acute angles such that sinB = sinQ, then prove that $\angle B = \angle Q$.

Sol. Consider two right ΔABC and ΔPQR such that sin B = sinQ.

We have,



$\sin B = \frac{AC}{AB}$ and, $\sin Q = \frac{PR}{PQ}$

$\because$ $\sin B = \sin Q$

$\Rightarrow$ $\frac{AC}{AB} = \frac{PR}{PQ}$

$\Rightarrow$ $\frac{AC}{PR} = \frac{AB}{PQ} = k$, (say) ...(i)

$\Rightarrow$ $AC = k\,PR$ and $AB = k\,PQ$ ...(ii)

Using Pythagoras theorem in triangles ABC and PQR, we have

$AB^2 = AC^2 + BC^2$ and $PQ^2 = PR^2 + QR^2$

$\Rightarrow$ $BC = \sqrt{AB^2 - AC^2}$ and $QR = \sqrt{PQ^2 - PR^2}$

$\Rightarrow$ $\frac{BC}{QR} = \frac{\sqrt{AB^2 - AC^2}}{\sqrt{PQ^2 - PR^2}} = \frac{\sqrt{k^2PQ^2 - k^2PR^2}}{\sqrt{PQ^2 - PR^2}}$ [Using (ii)]

$\Rightarrow$ $\frac{BC}{QR} = \frac{k\sqrt{PQ^2 - PR^2}}{\sqrt{PQ^2 - PR^2}} = k$ ...(iii)

From (i) and (iii), we have,

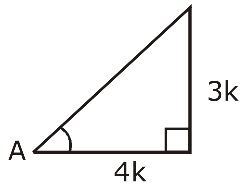
$\frac{AC}{PR} = \frac{AB}{PQ} = \frac{BC}{QR}$

$\Rightarrow$ $\Delta ACB \sim \Delta PRQ$

$\Rightarrow$ $\angle B = \angle Q$

Ex.7 If 3 cot A = 4, check whether $\frac{1 - \tan^2 A}{1 + \tan^2 A}$ = $\cos^2 A - \sin^2 A$ or not.

Sol. Here, 3 cot A = 4 or cot A = $\frac{4}{3}$



$\cot A = \frac{4}{3} = \frac{\text{base}}{\text{perpendicular}}$.

Let base = 4k, perpendicular = 3k

Using Pythagoras Theorem

$(\text{hyp.})^2 = (3k)^2 + (4k)^2 = 9k^2 + 16k^2 = 25k^2$

$\Rightarrow$ hyp. = $\sqrt{25k^2} = 5k$

So $\tan A = \frac{\text{perpendicular}}{\text{base}} = \frac{3k}{4k} = \frac{3}{4}$

$\cos A = \frac{\text{base}}{\text{hypotenuse}} = \frac{4}{5}$

$\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{3}{5}$

$\therefore$ L.H.S.

$$= \frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{\frac{16-9}{16}}{\frac{16+9}{16}} = \frac{\frac{7}{16}}{\frac{25}{16}}$$

$$= \frac{7}{16} \times \frac{16}{25} = \frac{7}{25}$$

R.H.S. = $\cos^2 A - \sin^2 A$

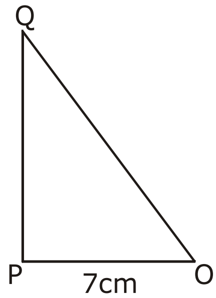
$$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

$\therefore$ L.H.S. = R.H.S.

$\therefore$ $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$

Ex.8 In ΔOPQ right angled at P, OP = 7cm, OQ – PQ = 1 cm. Determine the value of sin Q and cos Q.

Sol. In ΔOPQ



By Pythagoras Theorem

$OQ^2 = OP^2 + PQ^2$

$\Rightarrow$ $(1 + PQ)^2 = OP^2 + PQ^2$

($\because$ OQ – PQ = 1, OQ = 1 + PQ)

$\Rightarrow$ $1 + PQ^2 + 2PQ = OP^2 + PQ^2$

(Using $(a + b)^2 = a^2 + b^2 + 2ab$)

$\Rightarrow$ $1 + 2PQ = OP^2$

$\Rightarrow$ $1 + 2PQ = (7)^2$

$\Rightarrow$ $1 + 2PQ = 49$

$\Rightarrow$ $2PQ = 49 - 1 = 48$

$\Rightarrow$ $PQ = \frac{48}{2} = 24cm$

$\Rightarrow$ $OQ = 1 + PQ = 1 + 24\text{ cm} = 25cm$

So, $\sin Q = \frac{7}{25}$ and $\cos Q = \frac{24}{25}$

Ex.9 Choose the correct option and justify

(i) $\frac{2\tan 30^\circ}{1+\tan^2 30}$ is equal to

(A) sin 60° (B) cos 60° (C) tan 60° (D) sin 30°

(ii) $\frac{1-\tan^2 45}{1+\tan^2 45}$ is equal to

(A) tan 90° (B) 1 (C) sin 45° (D) 0

Sol. (i) $\frac{2\tan 30^\circ}{1+\tan^2 30}$

$$= \frac{2\times\frac{1}{\sqrt{3}}}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{3+1}{3}}$$

$$= \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}} = \frac{2}{\sqrt{3}}\times\frac{3}{4} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$= \frac{\sqrt{3}}{2} = \sin 60^\circ$. **Ans (A) = sin 60°.**

(ii) $\frac{1-\tan^2 45}{1+\tan^2 45} = \frac{1-(1)^2}{1+(1)^2} = \frac{1-1}{1+1} = \frac{0}{2} = 0$ **Ans. D = 0.**

Ex.10 If $\sin(A - B) = \frac{1}{2}$, $\cos(A + B) = \frac{1}{2}$, $0 < A + B \le 90^\circ$ find A and B.

Sol. $\sin(A - B) = \frac{1}{2}$

$\Rightarrow$ $A - B = 30^\circ$ .... (i) [Q $\sin 30^\circ = \frac{1}{2}$]

$\cos(A + B) = \frac{1}{2}$,

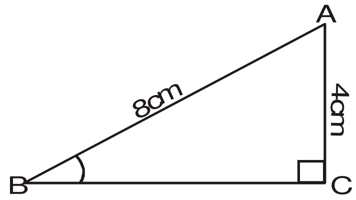
$\Rightarrow$ $A + B = 60^\circ$ ....(ii) [Q $\cos 60^\circ = \frac{1}{2}$]

Solving (i) and (ii)

$A = 45^\circ$ and $B = 15^\circ$

Ex.11 In ΔABC, right angled at C, if AC = 4 cm and AB = 8 cm. Find $\angle A$ and $\angle B$.

Sol. We are given, AC = 4 cm and AB = 8 cm



Now $\sin B = \frac{AC}{AB} = \frac{4}{8} = \frac{1}{2}$

But we know that $\sin 30° = \frac{1}{2}$

$\therefore$ B = 30°

Now $\angle A = 90° - \angle B$ ...[$\because \angle A + \angle B = 90°$]

$= 90° - 30° = 60°$

Hence, $\angle A = 60°$ and $\angle B = 30°$.

Ex.12 Evaluate : $\frac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° + \cos 60° + \cot 45°}$

Sol. $$\frac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° + \cos 60° + \cot 45°} = \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1}$$

$$= \frac{\frac{\sqrt{3} + 2\sqrt{3} - 4}{2\sqrt{3}}}{\frac{4 + \sqrt{3} + 2\sqrt{3}}{2\sqrt{3}}}$$

$$= \frac{3\sqrt{3} - 4}{3\sqrt{3} + 4} = \frac{3\sqrt{3} - 4}{3\sqrt{3} + 4} \times \frac{(3\sqrt{3} - 4)}{(3\sqrt{3} - 4)}$$

$$= \frac{27 + 16 - 24\sqrt{3}}{27 - 16} = \frac{43 - 24\sqrt{3}}{11}$$

Ex.13 Show that

(i) $\tan 10^o \tan 15^o \tan 75^o \tan 80^o = 1$

(ii) $\cos 1^o \cos 2^o \cos 3^o \ldots\ldots \cos 180^o = 0$

Sol. (i) L.H.S. $= \tan 10^o \tan 15^o \tan 75^o \tan 80^o$

$= \tan (90^o - 80^o) \tan (90^o - 75^o) \tan 75^o \tan 80^o$

$= \cot 80^o \cot 75^o \tan 75^o \tan 80^o$

$= (\cot 80^o \tan 80^o) (\cot 75^o \tan 75^o)$

$$= \left(\frac{1}{\tan 80^o}.\tan 80^o\right)\left(\frac{1}{\tan 75^o}.\tan 75^o\right)$$

$= 1 \times 1 = 1$ = R.H.S.

(ii) L.H.S.

$= \cos 1^o \cos 2^o \cos 3^o \ldots\ldots \cos 180^o$

$= \cos 1^o \cos 2^o \cos 3^o \ldots\ldots \cos 89^o \cos 90^o$

$\cos 91^o \ldots\ldots \cos 180^o = 0$ = R.H.S.

Ex.14 If $\tan 2\theta = \cot(\theta + 6^o)$ where 2θ and $\theta + 6^o$ are acute angles, find the value of θ.

Sol. We have

$\tan 2\theta = \cot(\theta + 6^o)$

or $\cot(90 - 2\theta) = \cot(\theta + 6^o)$

$90^o - 2\theta = \theta + 6^o$

$3\theta = 84^o$

Hence, $\theta = 28^o$

Ex.15 Without using tables, show that $(\cos 35° \cos 55° - \sin 35° \sin 55°) = 0$.

Sol. LHS $= (\cos 35° \cos 55° - \sin 35° \sin 55°)$

$= [(\cos 35° \cos 55° - \sin(90° - 55°) \sin(90° - 35°)]$

$= (\cos 35° \cos 55° - \cos 55° \cos 35°) = 0 =$ RHS.

$[\because \sin(90° - \theta) = \cos\theta$ and $\cos(90° - \theta) = \sin\theta]$

Ex.16 Express $(\sin 85° + \text{cosec } 85°)$ in terms of trigonometric ratios of angles between 0° and 45°.

Sol. $(\sin 85° + \text{cosec } 85°) = \sin(90° - 5°) + \text{cosec}(90° - 5°) = (\cos 5° + \sec 5°)$.

Ex.17 If $\tan 2A = \cot(A - 18°)$, where 2A is an acute angle, find the value of A.

Sol. We are given,

$\tan 2A = \cot(A - 18°)$

or $\cot(90° - 2A) = \cot(A - 18°)$ ...$[\because \cot(90° - 2A) = \tan 2A]$

$\therefore$ $90° - 2A = A - 18°$

or $A + 2A = 90° - 18°$

or $3A = 108°$

$\therefore$ $A = 36°$

Ex.18 Evaluate : $\frac{\sec 29°}{\text{cosec } 61°} + 2 \cot 8° \cot 17° \cot 45° \cot 73° \cot 82°$.

Sol. $\frac{\sec 29°}{\text{cosec } 61°} + 2 \cot 8° \cot 17° \cot 45° \cot 73° \cot 82°$.

$= \frac{\sec 29°}{\text{cosec }(90° - 29°)} + 2 \cot 8° \cot 17° (1) \cot(90° - 17°) \cot(90° - 8°)$

$= \frac{\sec 29°}{\sec 29°} + 2 \cot 8° \cot 17° \tan 17° \tan 8°$. $[\because \cot(90° - \theta) = \tan\theta]$

$= 1 + 2 \cot 8° \cot 17° \cdot \frac{1}{\cot 17°} \cdot \frac{1}{\cot 8°}$ $\ldots\left(\because \tan\theta = \frac{1}{\cot\theta}\right)$

$= 1 + 2 = 3$.

Ex.19 For a triangle ABC, show that $\sin\left(\frac{B+C}{2}\right) = \cos\left(\frac{A}{2}\right)$, where A, B and C are interior angles of ΔABC.

Sol. We know that $\angle A + \angle B + \angle C = 180°$

Thus we have, $B + C = 180° - A$

or $\frac{B+C}{2} = 90° - \frac{A}{2}$ or $\sin\left(\frac{B+C}{2}\right) = \sin\left(90° - \frac{A}{2}\right)$ or $\sin\left(\frac{B+C}{2}\right) = \cos\left(\frac{A}{2}\right)$.

Ex.20 Show that

$$\frac{1}{\text{cosec } A - \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\text{cosec } A + \cot A}$$

Sol. The question can be rewritten as

$$\frac{1}{\text{cosec } A - \cot A} + \frac{1}{\text{cosec } A + \cot A}$$

$$= \frac{1}{\sin A} + \frac{1}{\sin A} = \frac{2}{\sin A}$$

Now, L.H.S. $= \frac{1}{\text{cosec } A - \cot A} + \frac{1}{\text{cosec } A + \cot A}$

$$= \frac{\text{cosec } A + \cot A + \text{cosec } A - \cot A}{\text{cosec}^2 A - \cot^2 A} = \frac{2\text{cosec } A}{1}$$

$[\because \text{cosec}^2 A - \cot^2 A = 1]$

$$= \frac{2}{\sin A} = \text{R.H.S.}$$

Ex.21 Prove the following identities

(i) $\frac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = \frac{1 + \sin A}{\cos A}$

(ii) $(1 + \cot A + \tan A)(\sin A - \cos A)$

$$= \frac{\sec A}{\text{cosec}^2 A} - \frac{\text{cosec } A}{\sec^2 A}$$

Sol. (i) L.H.S. $= \frac{\tan A + \sec A - 1}{\tan A - \sec A + 1}$

$$= \frac{(\tan A + \sec A) - (\sec^2 A - \tan^2 A)}{\tan A - \sec A + 1} \quad [\because \sec^2 A - \tan^2 A = 1]$$

$$= \frac{(\tan A + \sec A) - (\sec A + \tan A)(\sec A - \tan A)}{\tan A - \sec A + 1}$$

$$= \frac{(\tan A + \sec A)(1 - \sec A + \tan A)}{\tan A - \sec A + 1}$$

$$= \tan A + \sec A = \frac{\sin A}{\cos A} + \frac{1}{\cos A}$$

$$= \frac{\sin A + 1}{\cos A} = \frac{1 + \sin A}{\cos A} = \text{R.H.S.}$$

(ii) L.H.S. = (1 + cot A + tan A) (sin A – cos A)

$= \left(1 + \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}\right)(\sin A - \cos A)$

$= \left(\frac{\sin A \cos A + \cos^2 A + \sin^2 A}{\sin A \cos A}\right)(\sin A - \cos A)$

$= \frac{(\sin A - \cos A)(\sin^2 A + \cos^2 A + \sin A \cos A)}{\sin A \cos A}$

$= \frac{\sin^3 A - \cos^3 A}{\sin A \cos A}$

[$\because$ (a – b) $(a^2 + b^2 + ab) = a^3 - b^3$]

$= \frac{\sin^3 A}{\sin A \cos A} - \frac{\cos^3 A}{\sin A \cos A}$ $= \frac{\sin^2 A}{\cos A} - \frac{\cos^2 A}{\sin A}$

$= \frac{\frac{1}{\cos A}}{\frac{1}{\sin^2 A}} - \frac{\frac{1}{\sin A}}{\frac{1}{\cos^2 A}}$ $= \frac{\sec A}{\operatorname{cosec}^2 A} - \frac{\operatorname{cosec} A}{\sec^2 A}$ = R.H.S.

Ex.22 If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, show that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.

Sol. We have $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

$\Rightarrow$ $(\cos\theta + \sin\theta)^2 = (\sqrt{2}\cos\theta)^2$ [squaring both sides]

$\Rightarrow$ $\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2\cos^2\theta$

$\Rightarrow$ $\sin^2\theta = 2\cos^2\theta - \cos^2\theta - 2\sin\theta\cos\theta$

$\Rightarrow$ $\sin^2\theta = \cos^2\theta - 2\sin\theta\cos\theta$

$\Rightarrow$ $\sin^2\theta + \sin^2\theta = \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta$

$\Rightarrow$ $2\sin^2\theta = (\cos\theta - \sin\theta)^2$

$\Rightarrow$ $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$ [assuming $\cos\theta > \sin\theta$]

ALTER :

$\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

$\Rightarrow$ $(\cos\theta + \sin\theta)^2 = (\sqrt{2}\cos\theta)^2$

$\Rightarrow$ $\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta$

$= 2\cos^2\theta \Rightarrow \cos^2\theta - \sin^2\theta = 2\sin\theta\cos\theta$

$\Rightarrow$ $(\cos\theta + \sin\theta)(\cos\theta - \sin\theta) = 2\sin\theta\cos\theta$

$\Rightarrow$ $\cos\theta - \sin\theta = \frac{2\sin\theta\cos\theta}{\cos\theta + \sin\theta}$

$\Rightarrow$ $\cos\theta - \sin\theta = \frac{2\sin\theta\cos\theta}{\sqrt{2}\cos\theta}$

[$\because$ $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$]

$\Rightarrow$ $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

The identity $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$ is true only if $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$ is true, these types of identities are called conditional identities.

Ex.23 Express sin A, sec A and tan A in terms of cot A.

Sol. We know that

$$\sin A = \frac{1}{\operatorname{cosec} A} = \frac{1}{\sqrt{\operatorname{cosec}^2 A}} = \frac{1}{\sqrt{1+\cot^2 A}}$$

$$\sec A = \frac{1}{\cos A} = \frac{\frac{1}{\sin A}}{\frac{\cos A}{\sin A}}$$

...(Dividing num. and deno. by sin A)

$$= \frac{\operatorname{cosec} A}{\cot A} = \frac{\sqrt{1+\cot^2 A}}{\cot A} \text{ and } \tan A = \frac{1}{\cot A}$$

Ex.24 Prove $\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta} = \tan\theta + \cot\theta$

Sol. LHS $= \sqrt{\sec^2\theta + \operatorname{cosec}^2\theta} = \sqrt{(1+\tan^2\theta)+(1+\cot^2\theta)} = \sqrt{\tan^2\theta + \cot^2 + 2\tan\theta\cot\theta}$

$= \sqrt{(\tan\theta+\cot\theta)^2} = \tan\theta + \cot\theta =$ RHS

Hence, proved.

Ex.25 Prove (cosecA – sinA) (sec A – cos A) $= \frac{1}{\tan A + \cot A}$

Sol. LHS = (cosecA – sinA) (sec A – cos A) $= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$

$= \left(\frac{1-\sin^2 A}{\sin A}\frac{1-\cos^2 A}{\cos A}\right) = \frac{\cos^2 A}{\sin A}\frac{\sin^2 A}{\cos A}$ $[\because \sin^2 A + \cos^2 A = 1]$

$= \sin A \cos A = \frac{\sin A \cos A}{\sin^2 A + \cos^2 A}$ $[\because \sin^2 A + \cos^2 A = 1]$

$= \frac{\frac{\sin A\cos A}{\sin A \cos A}}{\frac{\sin^2 A}{\sin A\cos A} + \frac{\cos^2 A}{\sin A \cos A}}$ [Dividing the numerator and denominator by sin A cos A.]

$= \frac{1}{\tan A + \cot A}$ = RHS Hence, proved.

Ex.26 If x = a sin θ and y = b tan θ, prove that $\left(\frac{a^2}{x^2} - \frac{b^2}{y^2}\right) = 1$

Sol. We have :

L.H.S. $= \frac{a^2}{x^2} - \frac{b^2}{y^2} = \frac{a^2}{a^2\sin^2\theta} - \frac{b^2}{b^2\tan^2\theta}$

$= \frac{1}{\sin^2\theta} - \frac{1}{\tan^2\theta} = \operatorname{cosec}^2 - \cot^2\theta$

= 1 = R.H.S.

$[\because 1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$\Rightarrow \operatorname{cosec}^2\theta - \cot^2\theta = 1]$

Ex.27 If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, show that $m^2 - n^2 = 4\sqrt{mn}$ = (or) $(m^2 - n^2) = 16mn$

Sol. L.H.S. $= m^2 - n^2$

$= (\tan\theta + \sin\theta) - (\tan\theta - \sin\theta)^2$

$[\because\ m = \tan\theta + \sin\theta \text{ and } n = \tan\theta - \sin\theta]$

$= (\tan^2\theta + \sin^2\theta + 2\tan\theta\sin\theta) - (\tan^2\theta + \sin^2\theta - 2\tan\theta\sin\theta)$

$= 4\tan\theta\sin\theta$

And, R.H.S.

$= 4\sqrt{mn} = 4\sqrt{(\tan\theta + \sin\theta)(\tan\theta - \sin\theta)}$

$= 4\sqrt{\tan^2\theta - \sin^2\theta} = 4\sqrt{\dfrac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta}$

$= 4\sqrt{\dfrac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}} = 4\sqrt{\dfrac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta}}$

$= 4\sqrt{\dfrac{\sin^4\theta}{\cos^2\theta}} = 4\dfrac{\sin^2\theta}{\cos\theta} = 4\sin\theta\dfrac{\sin\theta}{\cos\theta}$

$= 4\sin\theta\tan\theta$

Thus, L.H.S. = R.H.S.

i.e., $m^2 - n^2 = 4\sqrt{mn}$

Ex.28 If $\cot\theta = \dfrac{7}{8}$, evaluate :

(i) $\dfrac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ (ii) $\cot^2\theta$

Sol. (i) $\dfrac{1-\sin^2\theta}{1-\cos^2\theta} = \dfrac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta = \dfrac{49}{64}$

(ii) $\left(\dfrac{7}{8}\right)^2 = \dfrac{49}{64}$

EXERCISE - I UNSOLVED PROBLEM

Q.1 Consider ΔACB, right-angled at C, in which AB = 29 units, BC = 21 units and $\angle ABC = \theta$. Determine the value of .

(i) $\cos^2\theta + \sin^2\theta$ (ii) $\cos^2\theta - \sin^2\theta$

Q.2 In ΔABC, B is a right anlge, AB = 8 cm and BC = 15 cm. If $\angle A = \alpha$ and $\angle C = \beta$, find $\sin\alpha$, $\cos\alpha$, $\tan\alpha$, $\sin\beta$, $\cos\beta$, $\tan\beta$.

Q.3 ΔABC has a right-angled at A. Write down sin B, cos C and tanB, if AB = 5 units, AC = 12 units and BC = 13 units.

Q.4 PQRS is a rectangle in which PQ = 12 units, QR = 16 units. If P is joined to R, find $\sin\angle RPQ$, $\cos\angle RPQ$ and $\tan\angle RPQ$.

Q.5 Given $\tan A = \frac{4}{3}$, find the other trigonometric ratios of the angle A.

Q.6 If tanC = 11, find sin C and cosC.

Q.7 If $\sin A = \frac{2}{3}$; find the other two ratios i.e., cos A and tan A.

Q.8 If $\tan\theta = \frac{4}{3}$, then prove than:

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \frac{1}{3}$$

Q.9 If $\sin P = \frac{3}{5}$ and $\angle P$ is an acute angle, then show that :

4 tan P + 3 sin P = 6 cosP.

Q.10 If $\tan\theta = \frac{20}{21}$, then show that :

$$\frac{\cos\theta - \sin\theta + 1}{\cos\theta + 1 + \sin\theta} = \frac{3}{7}$$

Q.11 If $\sin\theta = \frac{2mn}{m^2+n^2}$, then show that :

$$\frac{\sin\theta.\cot\theta}{\cos\theta} = 1.$$

Q.12 Evaluate the following:

(i) $\tan^2 60° + 2\sin^2 30° - 3\cos^2 45°$

(ii) cos45° cos30° – sin 45° sin30°

Q.13 If $2\sin 2\theta = \sqrt{3}$, find the value of θ.

Q.14 If sin (A + B) = 1 and $\cos(A-B) = \frac{\sqrt{3}}{2}$, then find A and B.

Q.15 Evaluate:

(i) tan 7° tan 23° tan 60° tan 67° tan 83°

(ii) sin 35° sin 55° – cos 35° cos55°

Q.16 Evaluate : $\frac{3\sin 72°}{\cos 18°} - \frac{\sec 32°}{\operatorname{cosec} 58°}$

Q.17 Express each one of the following in terms of trigonometric ratios of angles lying between 0° and 45°.

(i) cos 56° + cot 56°

(ii) cot 75° + cos 75°

(iii) tan 68° + sec 68°

(iv) cosec 85° + sin85°

Q.18 If P, Q and R the interior angles of a triangle PQR, prove that:

$$\tan\left(\frac{Q+R}{2}\right) = \cot\left(\frac{P}{2}\right)$$

Q.19 If $\sin\alpha = \cos\beta$, then prove that

$\alpha + \beta = 90°$

Q.20 If $\cot 5\alpha = \tan(\alpha - 18°)$, where 5α is an acute angle, find the value of α.

Q.21 Evaluate :

$$\left[\frac{\sin 35°}{\cos 55°}\right]^2 + \left[\frac{\cos 55°}{\sin 35°}\right]^2 - 2\cos 60°$$

Q.22 Evaluate :

(i) $\dfrac{\cot 54°}{\tan 36°} + \dfrac{\tan 20°}{\cot 70°} - 2\tan 45°$

(ii) $\cos(70° + \theta) - \sin(20° - \theta)$

Q.23 Express the ratios cos A, tan A and sec A in terms of sin A.

Q.24 Write all the other trigonometric ratios of $\angle A$ in terms of cosA.

Q.25 Prove $\dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta$

Q.26 Prove $\dfrac{\cot\theta}{\operatorname{cosec}\theta+1} + \dfrac{\operatorname{cosec}\theta+1}{\cot\theta} = 2\sec\theta$

Q.27 Prove $\dfrac{\cos A}{1-\sin A} + \dfrac{\sin A}{1-\cos A} + 1$

$= \dfrac{\sin A\cos A}{(1-\sin A)(1-\cos A)}$

Q.28 Prove $(\sin A + \operatorname{cosec} A)^2 +$

$(\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

Q.29 Prove $(\sec A + \tan A - 1)$

$(\sec A - \tan A + 1) = \dfrac{2}{\cot A}$

Q.30 Prove $\sqrt{\dfrac{1-\cos A}{1+\cos A}} = \dfrac{\sin A}{1+\cos A}$

Q.31 Prove $\sqrt{\dfrac{1+\sin^2\theta.\sec^2\theta}{1+\cos^2\theta.\operatorname{cosec}^2\theta}} = \tan\theta$

Q.32 Prove $\sqrt{\sec^2 + \operatorname{cosec}^2\theta} = \tan\theta + \cot\theta$

Q.33 Prove $\sqrt{\dfrac{1+\sin A}{1-\sin A}} = \dfrac{1+\sin A}{\cos A}$

Q.34 Prove $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}} - \sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}} = \dfrac{2}{\cot\theta}$

Q.35 Prove $\dfrac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1} = \dfrac{\cot A - \cos A}{\cot A + \cos A}$

Q.36 Prove $\dfrac{\tan\theta + \sec\theta}{\tan\theta - \sec\theta} = \dfrac{\sin\theta + 1}{\sin\theta - 1}$

Q.37 Prove $\dfrac{\cot\theta + \operatorname{cosec}\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1} = \dfrac{1+\cos\theta}{\sin\theta}$

Q.38 Prove $\dfrac{\operatorname{cosec} A}{\operatorname{cosec} A + 1} = \dfrac{1-\sin A}{\cos^2 A}$

Q.39 Prove $(\sec\theta - \tan\theta)^2 = \dfrac{1-\sin\theta}{1+\sin\theta}$

Q.40 Prove $(\sec A + \tan A)^2 = \dfrac{1+\sin A}{1-\sin A}$

Q.41 Prove $(1 - \sin A + \cos A)^2$

$= 2(1 - \sin A)(1 + \cos A)$

Q.42 Prove $\dfrac{2\cos^2\theta - 1}{\sin\theta\cos\theta} = \cot\theta - \tan\theta$

Q.43 Prove $\frac{\tan^3\theta}{1+\tan^2\theta}+\frac{\cot^3\theta}{1+\cot^2\theta}$

$= \sec\theta \operatorname{cosec}\theta - 2\sin\theta\cos\theta$

Q.44 Prove

$$\left[\frac{1}{\sec^2\theta-\cos^2\theta}+\frac{1}{\operatorname{cosec}^2\theta-\sin^2\theta}\right]$$

$$\sin^2\theta\cos^2\theta = \frac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta}$$

Q.45 Prove $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta.\cos^2\theta$

Q.46 Prove $\sin^2\theta\tan\theta + \cos^2\theta\cot\theta + 2\sin\theta\cos\theta = \tan\theta + \cot\theta$

Q.47 Prove $\tan^2\theta + \cot^2\theta + 2 = \sec^2\theta\operatorname{cosec}^2\theta$

Q.48 Prove $(\sin\theta + \sec\theta)^2 + (\cos\theta + \operatorname{cosec}\theta)^2$

$= (1 + \sec\theta\operatorname{cosec}\theta)^2$.

Q.49 Prove $2\sec^2\theta - \sec^4\theta - 2\operatorname{cosec}^2\theta + \operatorname{cosec}^4\theta = \cot^4\theta - \tan^4\theta$

Q.50 Prove $\sin\theta(1+\tan\theta) + \cos\theta(1+\cot\theta)$

$= \sec\theta + \operatorname{cosec}\theta$

Q.51 In a right triangle ABC, right-angled at B, if tan A = 1, then verify that 2 sin A cos A = 1

Q.52 If A, B and C are interior angles of a triangle ABC, then show that $\sin\left[\frac{B+C}{2}\right]=\cos\frac{A}{2}$.

Q.53 Prove that sec A (1 − sin A)(sec A + tan A) = 1.

Q.54 Prove that $\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}=\frac{1}{\sec\theta-\tan\theta}$, using the identity $\sec^2\theta = 1 + \tan^2\theta$.

Q.55 Prove that

(i) $\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=2\sec A$

(ii) $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\operatorname{cosec}\theta$

(iii) $\frac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta}=\tan\theta$

(iv) $\left[\frac{1+\tan^2 A}{1+\cot^2 A}\right]=\left[\frac{1-\tan A}{1-\cot A}\right]^2=\tan^2 A$

Q.56 If $\cos\theta=\frac{4}{5}$, find all other trigonometric ratios of angle θ.

Q.57 If $\sin\theta=\frac{1}{\sqrt{2}}$, find all other trigonometric ratios of angle θ.

Q.58 If $\tan\theta = 8/15$ and $0° < \theta < 90°$, find $\sin\theta$

Q.59 If $\sin\theta = 8/17$ and $0° < \theta < 90°$, find $\tan\theta$

Q.60 If $\sin A=\frac{24}{25}$, find the value of tan A + sec A, where $0° < A < 90°$.

Q.61 If tan A = 1 and $\sin B=\frac{1}{\sqrt{2}}$, find the value of cos(A + B), if A and B are both acute angles.

Q.62 If $\tan A=\frac{1}{2}$ and $\tan B=\frac{1}{3}$, by using tan(A + B) $=\frac{\tan A+\tan B}{1-\tan A\tan B}$, prove that A + B = 45°.

Q.63 If 5 tanθ = 12, find the value of $\frac{2\cos\theta + \sin\theta}{\sin\theta - \cos\theta}$

Q.64 If $\tan\theta = \frac{1}{\sqrt{2}}$, find the value of $\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \cot^2\theta}$.

Q.65 If $\tan\theta = \frac{3}{4}$, find the value of $\frac{1-\cos\theta}{1+\cos\theta}$.

Q.66 If $\tan\theta = \frac{12}{5}$, find the value of $\frac{1+\sin\theta}{1-\sin\theta}$.

Q.67 If $\cot\theta = \frac{1}{\sqrt{3}}$, find the value of $\frac{1-\cos^2\theta}{2-\sin^2\theta}$

Q.68 If $\operatorname{cosec} A = \sqrt{2}$, find the value of $\frac{2\sin^2 A + 3\cot^2 A}{4[\tan^2 A - \cos^2 A]}$

Q.69 If $\sec\theta = \frac{13}{5}$, find the value of $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$.

Q.70 If 5 tanθ = 6, find the value of $\frac{8\sin\theta + 3\cos\theta}{8\sin\theta - 3\cos\theta}$.

Q.71 If 4 cotθ = 5, find the value of $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta}$.

Q.72 If sin A + cosec A = 3, find the value of $\frac{\sin^4 A + 1}{\sin^2 A}$

Q.73 If $\tan\theta = \frac{3}{4}$, find the value of $\frac{4\sin\theta - 2\cos\theta}{4\sin\theta + 3\cos\theta}$.

Q.74 If $\tan\theta = \frac{m}{n}$, find the value of $\frac{m\sin\theta + n\cos\theta}{m\sin\theta - n\cos\theta}$.

Q.75 If $\cot\theta = \sqrt{3}$, find the value of $\frac{\operatorname{cosec}^2\theta + \cot^2\theta}{\operatorname{cosec}^2\theta - \sec^2\theta}$.

Q.76 If sinθ – cosθ = 0 and 0° < θ < 90°, find the value of θ.

Q.77 If 3 cosθ = 1, find the value of $\frac{6\sin^2\theta + \tan^2\theta}{4\cos\theta}$.

Q.78 If 2cos(A + B) = 1 and 2 sin(A – B) = 1, find A, B.

Q.79 If tan(A + B) = 1 and sin(2A – B) = 1, find A, B.

Q.80 If 4 tanθ = 3, find the value of $\frac{4\sin\theta - 2\cos\theta}{4\sin\theta + 3\cos\theta}$.

Q.81 If $\sqrt{3}\tan\theta = 3\sin\theta$ find the value of $\sin^2\theta - \cos^2\theta$.

Q.82 If $\operatorname{cosec}\theta = \frac{13}{12}$, find the value of $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$.

ANSWER KEY

1. (i) 1 (ii) $\frac{41}{841}$

2. $\frac{15}{17}, \frac{8}{17}, \frac{15}{8}, \frac{8}{17}, \frac{15}{17}, \frac{8}{15}$

3. $\frac{12}{13}, \frac{12}{13}, \frac{12}{5}$

4. $\frac{4}{5}, \frac{3}{5}, \frac{4}{3}$

5. $\sin A = \frac{4}{5}$, $\cos A = \frac{3}{5}$, $\operatorname{cosec} A = \frac{5}{4}$,

$\sec A = \frac{5}{3}$, $\cot A = \frac{3}{4}$

6. $\frac{11}{\sqrt{122}}, \frac{1}{\sqrt{122}}$

7. $\frac{\sqrt{5}}{3}, \frac{2}{\sqrt{5}}$

12. (i) 2 (ii) $\frac{\sqrt{6}-\sqrt{2}}{4}$

13. 30º

14. 60º, 30º

15. (i) $\sqrt{3}$ (ii) 0

16. 2

17. (i) sin 34º + tan 34º (ii) tan 15º + sin 15º
(iii) cot 22º + cosec 22º (iv) sec 5º + cos 5º

20. 18º

21. 1

22. (i) 0 (ii) 0

23. $\cos A = \pm\sqrt{1-\sin^2 A}$, $\tan A = \pm\frac{\sin A}{\sqrt{1-\sin^2 A}}$,

$\sec A = \pm\frac{1}{\sqrt{1-\sin^2 A}}$

24. $\sin A = \pm\sqrt{1-\cos^2 A}$,

$\tan A = \pm\frac{\sqrt{1-\cos^2 A}}{\cos A}$,

$\operatorname{cosec} A = \pm\frac{1}{\sqrt{1-\cos^2 A}}$

$\sec A = \frac{1}{\cos A}$, $\cot A = \pm\frac{\cos A}{\sqrt{1-\cos^2 A}}$

56. $\sin\theta = 3/5$, $\tan\theta = 3/4$, $\sec\theta = 5/4$, $\operatorname{cosec}\theta = 5/3$, $\cot\theta = 4/3$

57. $\cos\theta = \frac{1}{\sqrt{2}}$, $\tan\theta = 1$, $\sec\theta = \sqrt{2}$, $\operatorname{cosec}\theta = \sqrt{2}$, $\cot\theta = 1$.

58. 8/17

59. 8/15

60. 7

61. 0

63. $\frac{22}{7}$

64. 3/10

65. $\frac{1}{9}$

66. 25

67. 3/5

68. 2

69. 3

70. $\frac{21}{11}$

71. 1/6

72. 7

73. 1/6

74. $\frac{m^2+n^2}{m^2-n^2}$

75. $\frac{21}{8}$

76. 45º

77. 10

78. A = 45º, B = 15º

79. A = 45º, B = 0º

80. $\frac{1}{6}$

81. $\frac{1}{3}$

82. 3

EXERCISE – II

BOARD PROBLEMS

Q.1 Without using tables, find the value of 14 sin 30° + 6 cos 60° – 5 tan 45°. **[ICSE-2004]**

Q.2 Prove that :

$$\frac{\operatorname{cosec}A}{\operatorname{cosec}A-1}+\frac{\operatorname{cosec}A}{\operatorname{cosec}A+1}=2+2\tan^2 A$$

[CBSE-AI-2004C]

Q.3 Evaluate :

$$\frac{\sec\theta\cdot\operatorname{cosec}(90^\circ-\theta)-\tan\theta\cot(90^\circ-\theta)+\sin^2 55^\circ+\sin^2 35^\circ}{\tan 10^\circ\tan 20^\circ\tan 60^\circ\tan 70^\circ\tan 80^\circ}.$$

[CBSE-AI-2004C]

Q.4 Without using mathematical tables, find the value of x if cos x = cos 60° cos 30° + sin 60° sin 30°. **[ICSE-2005]**

Q.5 Without using trigonometric tables, evaluate:

$$\frac{2\tan 53^\circ}{\cot 37^\circ}-\frac{\cot 80^\circ}{\tan 10^\circ}.$$ **[ICSE-2006]**

Q.6 Without using trigonometric tables, evaluate:

$$\frac{\sin 80^\circ}{\cos 10^\circ}+\sin 59^\circ\sec 31^\circ.$$ **[ICSE-2007]**

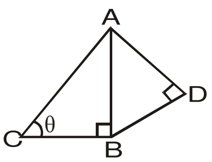
Q.7 Without using tables, evaluate

$$\frac{\sin 25^\circ}{\sec 65^\circ}+\frac{\cos 25^\circ}{\operatorname{cosec} 65^\circ}.$$ **[ICSE-2008]**

Q.8 Prove that $\frac{\sin A}{(1+\cos A)}=$ (cosec A – cot A).

[ICSE-2008]

Q.9 In the fig AD = 4 cm, BD = 3 cm and CB = 12 cm, find cot θ. **[CBSE-Delhi-2008]**



Q.10 Without using the trigonometric tables, evaluate the following :

$$\frac{11\sin 70^\circ}{7\cos 20^\circ}-\frac{4}{7}\frac{\cos 53^\circ\operatorname{cosec} 37^\circ}{\tan 15^\circ\tan 35^\circ\tan 55^\circ\tan 75^\circ}$$

[CBSE-Delhi-2008]

Q.11 Without using trigonometric tables, evaluate the following : $\frac{\sin 18^\circ}{\cos 72^\circ}+\sqrt{3}$ [tan 10° tan 30° tan 40° tan 50° tan 80°]. **[CBSE-Delhi-2008]**

Q.12 If sin θ = cos θ, find the value of θ. **[CBSE-AI-2008]**

Q.13 Without using trigonometric tables, evaluate the following : ($\sin^2 25^\circ+\sin^2 65^\circ$) + $\sqrt{3}$ (tan 5° tan 15° tan 30° tan 75° tan 85°). **[CBSE-AI-2008]**

Q.14 Without using trigonometric tables, evaluate the following : [$\cos^2 25^\circ+\cos^2 65^\circ$] + cosec θ sec (90° – θ) – cot θ tan (90° – θ). **[CBSE-AI-2008]**

Q.15 If 7 $\sin^2\theta$ + 3 $\cos^2\theta$ = 4 show that tan θ = $\frac{1}{\sqrt{3}}$. **[CBSE-AI-2008]**

Q.16 If tan A = $\frac{5}{12}$, find the value of (sin A + cos A) sec A. **[CBSE-Foreign-2008]**

Q.17 If sec 4A = cosec (A – 20°), where 4A is an acute angle, find the value of A. **[CBSE-Foreign-2008]**

OR

In a ΔABC, right angled at C, if tan A = $\frac{1}{\sqrt{3}}$, find the value of sin A cos B + cos A sin B.

Q.18 If cos A = $\frac{7}{25}$, find the value of tan A + cot A. **[CBSE-foreign-2008]**

Q.19 If sin θ = $\frac{1}{3}$, find the value of [2 $\cot^2\theta$ + 2] **[CBSE-Delhi-2009]**

Q.20 Simplify : $\frac{\sin^3\theta+\cos^3\theta}{\sin\theta+\cos\theta}$ + sin θ cos θ : **[CBSE-Delhi-2009]**

Q.21 If $\sec^2\theta$ (1 + sin θ) (1 – sin θ) = k, then find the value of k. **[CBSE-AI-2009]**

Q.22 If cot θ = $\frac{15}{8}$, then evaluate $\frac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)}$

OR

Find the value of tan 60° geometrically. **[CBSE-AI-2009]**

Q.23 If sec A = $\frac{15}{7}$ and A + B = 90°, find the value of cosec B. **[CBSE-foreign-2009]**

Q.24 Without using trigonometric tables, evaluate :

$$\frac{7\cos 70^\circ}{2\sin 20^\circ}+\frac{3}{2}\frac{\cos 55^\circ \operatorname{cosec} 35^\circ}{\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 85^\circ \tan 65^\circ}.$$

[CBSE-foreign-2009]

Q.25 Evaluate :

$$\frac{\sec^2 54^\circ - \cot^2 36^\circ}{\operatorname{cosec}^2 57^\circ - \tan^2 33^\circ}+2\sin^2 38^\circ \sec^2 52^\circ - \sin^2 45^\circ$$

[AI-2005]

Q.26 Prove the following : $(\tan A - \tan B)^2 + (1 + \tan A \tan B)^2 = \sec^2 A \sec^2 B$.

OR

Evaluate : $\sec^2 10^\circ - \cot^2 80^\circ +$

$$\frac{\sin 15^\circ \cos 75^\circ + \cos 15^\circ \sin 75^\circ}{\cos\theta \sin(90^\circ-\theta)+\sin\theta\cos(90^\circ-\theta)}.$$

[Foreign-2005]

Q.27 Prove that : $\frac{\sec\theta+\tan\theta-1}{\tan\theta-\sec\theta+1}=\frac{\cos\theta}{1-\sin\theta}$

OR

Without using trigonometric tables, evaluate the

following :

$$\frac{\cot(90^\circ-\theta).\sin(90^\circ-\theta)}{\sin\theta}+\frac{\cot 40^\circ}{\tan 50^\circ} - (\cos^2 20^\circ + \cos^2 70^\circ).$$

[Delhi-2005C]

Q.28 Without using trigonometric tables, evaluate the following :

$$\frac{\sec^2\theta - \cot^2(90^\circ-\theta)}{\operatorname{cosec}^2 67^\circ - \tan^2 23^\circ}+(\sin^2 40^\circ + \sin^2 50^\circ).$$

[AI-2005C]

Q.29 Prove that : $(1 + \tan A)^2 + (1 - \tan A)^2 = 2\sec^2 A$ **[ICSE-2005]**

Q.30 Prove that : $\frac{\sin\theta\tan\theta}{1-\cos\theta}=1+\sec\theta$.

[ICSE-2006]

Q.31 Prove that : $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}+\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}=\frac{2\sec^2\theta}{\tan^2\theta-1}$

[Delhi-2006]

OR

Evaluate without using trigonometric tables :

$$\frac{\sec^2(90^\circ-\theta)-\cot^2\theta}{2(\sin^2 25^\circ+\sin^2 65^\circ)}+\frac{2\cos^2 60^\circ \tan^2 28^\circ \tan^2 62^\circ}{3(\sec^2 43^\circ - \cot^2 47^\circ)}$$

Q.32 Prove that : $\frac{1}{\operatorname{cosec}\theta-\cot\theta}-\frac{1}{\sin\theta}$

$=\frac{1}{\sin\theta}-\frac{1}{\operatorname{cosec}\theta+\cot\theta}$ **[AI-2006]**

OR

Evaluate without using trigonometric tables :

$$\frac{\operatorname{cosec}^2(90^\circ-\theta)-\tan^2\theta}{4(\cos^2 48^\circ+\cos^2 42^\circ)}-\frac{2\tan^2 30^\circ \sec^2 52^\circ \sin^2 38^\circ}{\operatorname{cosec}^2 70^\circ - \tan^2 20^\circ}$$

Q.33 Evaluate without using trigonometric tables :

$$\frac{\sin^2\theta+\sin^2(90^\circ-\theta)}{3(\sec^2 61^\circ-\cot^2 29^\circ)}-\frac{3\cot^2 30^\circ \sin^2 54^\circ \sec^2 36^\circ}{2(\operatorname{cosec}^2 65^\circ-\tan^2 25^\circ)}$$

[Foreign-2006]

Q.34 Without using trigonometric tables, evaluate the following :

$\frac{\cos^2 20^\circ+\cos^2 70^\circ}{\sec^2 50^\circ-\cot^2 40^\circ} + 2\operatorname{cosec}^2 58^\circ - 2\cot 58^\circ \tan 32^\circ - 4\tan 13^\circ \tan 37^\circ \tan 45^\circ \tan 53^\circ \tan 77^\circ$

OR

Prove that : $\sqrt{\frac{\sec\theta-1}{\sec\theta+1}}+\sqrt{\frac{\sec\theta+1}{\sec\theta-1}} = 2\operatorname{cosec}\theta$

[Delhi-2006C]

Q.35 Without using trigonometric table, evaluate the following : **[Delhi-2007]**

(i) $\frac{\sec 39^\circ}{\operatorname{cosec} 51^\circ}+\frac{2}{\sqrt{3}}\tan 17^\circ \tan 38^\circ \tan 60^\circ \tan 52^\circ \tan 73^\circ - 3(\sin^2 31^\circ + \sin^2 59^\circ)$

[AI-2006C]

(ii)

$$\frac{3\cos 55^\circ}{7\sin 35^\circ}-\frac{4(\cos 70^\circ.\operatorname{cosec} 20^\circ)}{7(\tan 5^\circ.\tan 25^\circ.\tan 45^\circ.\tan 65^\circ.\tan 85^\circ)}$$

[Delhi-2007]

(iii) $\tan 7^\circ.\tan 23^\circ.\tan 60^\circ.\tan 67^\circ.\tan 83^\circ + \frac{\cot 54^\circ}{\tan 36^\circ} + \sin 20^\circ.\sec 70^\circ - 2$

[AI-2007]

Q.36 Prove that : $\frac{\sec A - 1}{\sec A + 1}=\frac{1-\cos A}{1+\cos A}$

[ICSE-2007]

Q.37 Prove that : $\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$

OR

Prove that : (1 + cot A – cosec A) (1 + tan A + sec A) = 2 **[CBSE (Delhi)-2008]**

Q.38 Prove that : $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$

OR

Prove that $\sin\theta\,(1 + \tan\theta) + \cos\theta\,(1 + \cot\theta) = \sec\theta + \operatorname{cosec}\theta$ **[CBSE-AI-2008]**

Q.39 Prove that : (1 + cot A + tan A) (sin A – cos A) = sin A tan A – cot A cost A.

[CBSE-foreign-2008]

OR

Without using trigonometric tables, evaluate the following :

$$2\left[\frac{\cos 58^\circ}{\sin 32^\circ}\right] - \sqrt{3}\left[\frac{\cos 38^\circ \operatorname{cosec} 52^\circ}{\tan 15^\circ \tan 60^\circ \tan 75^\circ}\right].$$

Q.40 Find the value of sin 30° geometrically.

OR

Without using trigonometrical tables, evaluate:

$$\frac{\cos 58^\circ}{\sin 32^\circ} + \frac{\sin 22^\circ}{\cos 68^\circ} -$$

$$\frac{\cos 38^\circ \operatorname{cosec} 52^\circ}{\tan 18^\circ \tan 35^\circ \tan 60^\circ \tan 72^\circ \tan 55^\circ}$$

[CBSE-Delhi-2009]

Q.41 Evaluate : $\frac{2}{3}\operatorname{cosec}^2 58^\circ - \frac{2}{3}\cot 58^\circ \tan 32^\circ - \frac{5}{3}$ tan13º tan37º tan45º tan53º tan77º.

[CBSE-AI-2009]

Q.42 Prove that : $\sec^2\theta - \frac{\sin^2\theta - 2\sin^4\theta}{2\cos^4\theta - \cos^2\theta} = 1$

[CBSE-foreign-2009]

ANSWER KEY

1.	5	**3.**	$\frac{2}{\sqrt{3}}$	**4.**	30°
5.	1	**6.**	2	**7.**	1
9.	$\frac{12}{5}$	**10.**	1	**11.**	2
12.	45°	**13.**	2	**14.**	2
16.	$\frac{17}{12}$	**17.**	22° or 1		
18.	$\frac{625}{168}$	**19.**	18	**20.**	1
21.	1	**22.**	$\frac{225}{64}$ or $\sqrt{3}$		
23.	$\frac{15}{7}$	**24.**	5	**25.**	$\frac{5}{2}$
26.	or 2	**27.**	or 1	**28.**	2
31.	or $\frac{2}{3}$	**32.**	or $\left[\frac{-5}{12}\right]$		
33.	$-\frac{25}{6}$	**34.**	1		
35.	(i) 0 (ii) $\frac{-1}{7}$ (iii) $\sqrt{3}-1$			**39.**	or 1
40.	$\frac{1}{2}$ or $\frac{2\sqrt{3}-1}{\sqrt{3}}$			**41.**	– 1

EXERCISE – III — MULTIPLE CHOICE QUESTIONS

Q.1 If tan A = $\frac{4}{3}$ and A is acute, then sin is equal to :

(A) $\frac{3}{5}$ (B) $\frac{4}{5}$

(C) $\frac{5}{3}$ (D) $\frac{5}{4}$

Q.2 If sin A = $\frac{8}{17}$ and A is acute, then cot A is equal to:

(A) $\frac{15}{8}$ (B) $\frac{15}{17}$

(C) $\frac{8}{15}$ (D) $\frac{17}{8}$

Q.3 $\frac{1-\tan^2 45^o}{1+\tan^2 45^o}$ is equal to

(A) tan 90º (B) 1
(C) sin 45º (D) 0

Q.4 sin 2A = 2 sin A is true, when A is equal to :

(A) 0º (B) 30º
(C) 45º (D) 60º

Q.5 $\frac{2\tan 30^o}{1+\tan^2 30^o}$ is equal to :

(A) sin 60º (B) cos 60º
(C) tan 60º (D) sin 30º

Q.6 $\frac{2\tan 30^o}{1-\tan^2 30^o}$ is equal to :

(A) cos 60º (B) sin 60º
(C) tan 60º (D) sin 30º

Q.7 If cosec A = $\frac{2}{\sqrt{3}}$ and A is acute, then tan A is equal to :

(A) $\frac{1}{\sqrt{3}}$ (B) $\frac{\sqrt{3}}{2}$

(C) $\sqrt{3}$ (D) 2

Q.8 If sin α = $\frac{1}{2}$ and α is acute, then (3 cos α – 4 cos^3 α) is equal to :

(A) 0 (B) $\frac{1}{2}$

(C) $\frac{1}{6}$ (D) –1

Q.9 $\frac{\tan 65^o}{\cot 25^o}$ is equal to :

(A) 0 (B) 1
(C) tan 65º (D) cot 25º

Q.10 If sin 3 A = cos(A – 26º), where 3A is an acute angle, then the value of A is :

(A) 26º (B) 23º
(C) 30º (D) 29º

Q.11 If tan A = cot B, then A + B is equal to

(A) 30º (B) 0º
(C) 60º (D) 90º

Q.12 (tan 7º. tan 83º) (tan 23º.tan 67º). tan 60º is equal to :

(A) 1 (B) 2

(C) $\sqrt{3}$ (D) $\frac{1}{\sqrt{3}}$

Q.13 If sin 5A = cos 4A, where 5A and 4A are acute angles, then value of A is :

(A) 10º (B) 20º
(C) 30º (D) 45º

Q.14 $\frac{\cos 70^o}{\sin 20^o}+\frac{\cos 59^o}{\sin 31^o}-8\sin^2 30^o$ is equal to :

(A) 1 (B) –1
(C) 0 (D) 2

Q.15 If 3 tan θ= 4, then $\frac{(3\sin\theta+2\cos\theta)}{3\sin\theta-2\cos\theta}$ is equal to:

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$

(C) 5 (D) 3

Q.16 If 5 cot θ = 3, then $\frac{(5\sin\theta-3\cos\theta)}{(\theta\sin\theta+3\cos\theta)}$ is equal to:

(A) $\frac{11}{18}$ (B) $\frac{16}{29}$

(C) $\frac{14}{27}$ (D) none of these

Q.17 If 3 cot θ = 5, then $\frac{(5\sin\theta-3\cos\theta)}{(5\sin\theta+3\cos\theta)}$ is equal to:

(A) $\frac{5}{3}$ (B) $\frac{3}{5}$

(C) 0 (D) 1

Q.18 $\frac{\cos\theta}{1-\tan\theta}+\frac{\sin\theta}{1-\cot\theta}$ is equal to :

(A) $\frac{1-\tan\theta}{1-\cos\theta}$ (B) $\frac{1-\cot\theta}{1-\tan\theta}$

(C) $\cos\theta + \sin\theta$ (D) none of these

Q.19 $\sec^2\theta + \text{cosec}^2\theta$ is equal to:

(A) $\frac{\sec^2\theta}{\text{cosec}^2\theta}$ (B) $\sec^2\theta - \text{cosec}^2\theta$

(C) $\frac{\cos^2\theta}{\sin^2\theta}$ (D) $\sec^2\theta \,.\, \text{cosec}^2\theta$

Q.20 $\sin\theta\cos(90° - \theta) + \cos\theta\sin(90° - \theta)$ is equal to:

(A) 0 (B) 1

(C) 2 (D) $\frac{3}{2}$

Q.21 $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$ is equal to :

(A) $\sin\theta$ (B) $\cos\theta$

(C) $\tan\theta$ (D) $\sec\theta$

Q.22 $\sec\theta(1 - \sin\theta)(\sec\theta + \tan\theta)$ is equal to :

(A) 0 (B) 1

(C) 2 (D) none of these

Q.23 $(\cos^4 x - \sin^4 x)$ is equal to :

(A) $2\sin^2 x - 1$ (B) $1 - 2\cos^2 x$

(C) $\sin^2 x - \cos^2 x$ (D) none of these

Q.24 $(\cos 0° + \sin 45° + \sin 30°)(\sin 90° - \cos 45° + \cos 60°)$ is equal to

(A) $\frac{3}{5}$ (B) $\frac{5}{6}$

(C) $\frac{7}{4}$ (D) $\frac{5}{8}$

Q.25 $\left(\frac{4}{3}\cot^2 30° + 3\sin^2 60° - 2\,\text{cosec}^2 60° - \frac{3}{4}\tan^2 30°\right)$ is equal to :

(A) $\frac{10}{3}$ (B) 3

(C) $\frac{8}{3}$ (D) $\frac{9}{4}$

Q.26 $\sqrt{\frac{\sec\theta-1}{\sec\theta+1}}+\sqrt{\frac{\sec\theta+1}{\sec\theta-1}}$ is equal to :

(A) $2\,\text{cosec}\,\theta$ (B) $\frac{2\sin\theta}{\sqrt{\sec\theta}}$

(C) $2\cos\theta$ (D) none of these

Q.27 $\tan 1° \tan 2° \tan 3° \ldots\ldots \tan 89°$ is equal to :

(A) 1

(B) 0

(C) cannot be determined

(D) none of these

Q.28 If $7\sin^2\theta + 3\cos^2\theta = 4$ and $0 \le \theta \le \frac{\pi}{2}$, then the value of $\tan\theta$ is :

(A) $\frac{1}{\sqrt{3}}$ (B) $\frac{1}{\sqrt{7}}$

(C) $\sqrt{\frac{2}{7}}$ (D) $\sqrt{\frac{3}{7}}$

Q.29 If $\sin\theta + \sin^2\theta = 1$, then $\cos^2\theta + \cos^4\theta$ is equal to :

(A) -1 (B) 1

(C) 0 (D) $\frac{1}{2}$

Q.30 If $x = r\sin\alpha\cos\beta$, $y = r\sin\alpha\sin\beta$ and $z = r\cos\alpha$, then :

(A) $x^2 + y^2 + z^2 = r^2$ (B) $x^2 + y^2 - z^2 = r^2$

(C) $x^2 - y^2 + z^2 = r^2$ (D) $x^2 + y^2 - z^2 = r^2$

Q.31 If $x = a\sin\theta$ and $y = b\tan\theta$, then $\frac{a^2}{x^2}-\frac{b^2}{y^2}$ is equal to :

(A) 0 (B) 1

(C) -1 (D) none of these

Q.32 If $(\cos\theta + \sin\theta) = 1$, then $(\cos\theta - \sin\theta)$ is equal to :

(A) 1 (B) -1

(C) ± 1 (D) 2

Q.33 $(\sin^6\theta + \cos^6\theta)$ is equal to :

(A) $3\sin^2\theta\cos^2\theta$ (B) $\sin^6\theta + \cos^6\theta$

(C) $\frac{3\sin^3\theta\cos^3\theta}{\text{cosec}\,\theta\,\sec\theta}$ (D) $1 - 3\sin^2\theta\cos^2\theta$

Q.34 If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, then $\cos\theta - \sin\theta$ is equal to :

(A) $\sqrt{2}\tan\theta$ (B) $\sqrt{2}\sin\theta$

(C) $\frac{\sqrt{2}}{\cos\theta + \sin\theta}$ (D) none of these

Q.35 If $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$, $\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta = 1$, then :

(A) $x^2 + y^2 = a^2 + b^2$

(B) $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 2$

(C) $a^2x^2 + b^2y^2 = 1$

(D) none of these

Q.36 If $\tan\alpha + \sin\alpha = m$ and $\tan\alpha - \sin\alpha = n$ then $m^2 - n^2$ is equal to :

(A) $\sqrt{mn}$ (B) $\sqrt{\frac{m}{n}}$

(C) $4\sqrt{mn}$ (D) none of these

Q.37 If $\tan A = n \tan B$ and $\sin A = m \sin B$, then $\frac{m^2-1}{n^2-1}$ is equal to :

(A) $\sin^2 A$ (B) $\frac{n^2-1}{m^2-1}$

(C) $\frac{m^2-1}{n^2-1}$ (D) none of these

Q.38 If $7\sin^2\theta + 3\cos^2\theta = 4$, then $\sec\theta + \operatorname{cosec}\theta$ is equal to

(A) $\frac{2}{\sqrt{3}} - 2$ (B) $\frac{2}{\sqrt{3}} + 2$

(C) $\frac{2}{\sqrt{3}}$ (D) none of these

Q.39 If θ is acute and $\frac{\cos^2\theta}{\cot^2\theta - \cos^2\theta} = 3$, then θ is equal to

(A) 60° (B) 30°
(C) 90° (D) none of these

Q.40 $\frac{\sin(90^\circ - \theta)\sin\theta}{\tan\theta} - 1$ is equal to :

(A) $-\cos^2\theta$ (B) $-\sin^2\theta$
(C) $1 - \sin^2\theta$ (D) $1 - \cos^2\theta$

Q.41 If θ is an acute angle and $\tan\theta + \cos\theta = 2$, then the value of $\tan^5\theta + \cot^5\theta$ is :

(A) 1 (B) 0
(C) –1 (D) 2

Q.42 The ratio of the length of a rod and its shadow is $1 : \sqrt{3}$. The altitude of the sun is :

(A) 30° (B) 45°
(C) 60° (D) 90°

Q.43 If A, B, C are interior angles of a ΔABC, then $\tan\left(\frac{B+C}{2}\right)$ is equal to :

(A) $\tan\frac{A}{2}$ (B) $\sin\frac{A}{2}$

(C) $\tan\frac{A-B}{2}$ (D) $\cot\frac{A}{2}$

Q.44 If $\sec\theta + \tan\theta = x$, then $\tan\theta$ is equal to :

(A) $\frac{x^2+1}{x}$ (B) $\frac{x^2-1}{x}$

(C) $\frac{x^2+2}{2x}$ (D) $\frac{x^2-1}{2x}$

Q.45 $\tan^2\theta \sin^2\theta$ is equal to

(A) $\tan^2\theta - \sin^2\theta$ (B) $\tan^2\theta + \sin^2\theta$

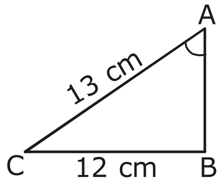
(C) $\frac{\tan^2\theta}{\sin^2\theta}$ (D) $\sin^2\theta \cot^2\theta$

Q.46 If ΔABC is right angled at B, AB = 3cm, BC = 4cm and AC = 5cm, then vlaue of cos C is

(A) $\frac{3}{4}$ (B) $\frac{5}{4}$

(C) $\frac{4}{3}$ (D) $\frac{4}{5}$

Q.47 In the adjoining figure, value of sin A is



(A) $\frac{5}{13}$ (B) $\frac{12}{5}$

(C) $\frac{12}{13}$ (D) $\frac{5}{12}$

Q.48 The value of cos 45° is

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{2}}{2}$

(C) $\frac{\sqrt{3}}{2}$ (D) $\frac{2}{\sqrt{3}}$

Q.49 The value of $\sin^2 30^\circ + \cos^2 30^\circ$ is

(A) $\frac{1}{4}$ (B) $\frac{3}{4}$

(C) 1 (D) $\frac{1}{2}$

Q.50 The value of $\sin^2 29^\circ + \sin^2 61^\circ$ is

(A) 1 (B) 0

(C) $\frac{1}{4}$ (D) $\frac{1}{2}$

Q.51 If $x = a\sin\theta + b\cos\theta$ and $y = a\cos\theta - b\sin\theta$ then $x^2 + y^2$ is equal to

(A) 1 (B) 2
(C) 2ab (D) $a^2 + b^2$

Q.52 If x = a cos θ and y = b sin θ, then $b^2x^2 + a^2y^2$ is equal to
(A) ab (B) a^2b^2
(C) a^4b^4 (D) $a^2 + b^2$

Q.53 If x = a sec θ and y = b tan θ, then $b^2x^2 - a^2y^2$ is equal to
(A) a^2b^2 (B) ab
(C) $a^2 - b^2$ (D) $a^2 + b^2$

Q.54 In a ΔABC, ∠B = 90º and tan A = $\frac{1}{\sqrt{3}}$, then value of sin A cos C + cos A sin C is
(A) $\frac{\sqrt{3}}{4}$ (B) $\frac{1}{4}$
(C) 1 (D) $\frac{\sqrt{3}}{2}$

Q.55 If $\tan\theta = \frac{2}{3}$, then value of $\sin^2\theta + \cos^2\theta$ is
(A) $\frac{5}{\sqrt{13}}$ (B) $\sqrt{13}$
(C) $\frac{5}{13}$ (D) 1

Q.56 If $\tan\theta = \frac{4}{3}$, the vlaue of $\frac{2\sin\theta - 3\cos\theta}{2\sin\theta + 3\cos\theta}$ is
(A) $\frac{1}{17}$ (B) $\frac{1}{51}$
(C) $-\frac{1}{17}$ (D) $-\frac{9}{17}$

Q.57 If 4 cot A = 3, then $\frac{\sin A - \cos A}{\sin A + \cos A}$ is equal to
(A) $\frac{5}{6}$ (B) $\frac{1}{7}$
(C) $\frac{6}{7}$ (D) $\frac{3}{4}$

Q.58 If $\tan\theta = \frac{p}{q}$, then value of $\frac{p\sin\theta - q\cos\theta}{p\sin\theta + q\cos\theta}$ is equal to
(A) $\frac{p^2 - q^2}{p^2 + q^2}$ (B) $\frac{p^2 + q^2}{p^2 - q^2}$
(C) $\frac{2pq}{p^2 + q^2}$ (D) $\frac{2pq}{p^2 - q^2}$

Q.59 The value of $\sin^2 60º - \sin^2 30º$ is
(A) $\frac{1}{4}$ (B) $\frac{1}{2}$
(C) $\frac{3}{4}$ (D) $-\frac{1}{2}$

Q.60 The value of sec 30º is
(A) 2 (B) $\frac{\sqrt{3}}{2}$
(C) $\frac{2}{\sqrt{3}}$ (D) $\sqrt{2}$

Q.61 $\frac{2\tan 60º}{1 + \tan^2 60º}$ is equal to
(A) sin 60º (B) sin 30º
(C) cos 60º (D) tan 60º

Q.62 If sec θ + tan θ = x, then sec θ is equal to
(A) $\frac{x^2 - 1}{2x}$ (B) $\frac{x^2 + 1}{2x}$
(C) $\frac{x^2 + 1}{x}$ (D) $\frac{x^2 - 1}{x}$

Q.63 If sin A + $\sin^2$ A = 1, then $\cos^2 A + \cos^4 A$ is equal to
(A) –1 (B) 1
(C) 2 (D) 0

Q.64 If cos θ + $\cos^2$ θ = 1, then $\sin^2\theta + \cos^4\theta$ is equal to
(A) –1 (B) 0
(C) 2 (D) 1

Q.65 9 $\sec^2$ A – 9 $\tan^2$ A is equal to
(A) 1 (B) –9
(C) 9 (D) 0

Q.66 $\cos^4 A - \sin^4 A$ is equal to
(A) $2\cos^2 A + 1$ (B) $2\cos^2 A - 1$
(C) $2\sin^2 A - 1$ (D) $2\sin^2 A + 1$

Q.67 $\sec^4 A - \sec^2 A$ is equal to
(A) $\tan^2 A - \tan^4 A$ (B) $\tan^4 A - \tan^2 A$
(C) $\sec^2 A.\text{cosec}^2 A$ (D) $\tan^2 A + \tan^4 A$

Q.68 The value of $\frac{\sin 36º}{2\cos 54º} - \frac{2\sec 41º}{3\,\text{cosec}\,49º}$ is
(A) –1 (B) $\frac{1}{6}$
(C) $-\frac{1}{6}$ (D) 1

Q.69 If sin (θ + 36º) = cos θ where θ + 36º is an acute angle, then vlaue of θ is
(A) 54º (B) 18º
(C) 21º (D) 27º

Q.70 $\dfrac{1+\tan^2\theta}{1+\cot^2\theta}$ equals
(A) $\cot^2\theta$ (B) $\sec^2\theta$
(C) –1 (D) $\tan^2\theta$

Q.71 sec θ is equal to
(A) $\dfrac{1}{\sqrt{1-\cos^2\theta}}$ (B) $\sqrt{\dfrac{1+\cot^2\theta}{\cot\theta}}$
(C) $\dfrac{\cot\theta}{\sqrt{1+\cot^2\theta}}$ (D) $\dfrac{\sqrt{\mathrm{cosec}^2\theta-1}}{\mathrm{cosec}\,\theta}$

Q.72 $\dfrac{\sin\theta}{1+\cos\theta}$ is equal to
(A) $\dfrac{1+\cos\theta}{\sin\theta}$ (B) $\dfrac{1-\cos\theta}{\cos\theta}$
(C) $\dfrac{1-\cos\theta}{\sin\theta}$ (D) $\dfrac{1-\sin\theta}{\cos\theta}$

Q.73 $\cos^2 37^\circ - \sin^2 53^\circ)$ = ?
(A) $\dfrac{1}{3}$ (B) $\dfrac{2}{\sqrt{3}}$
(C) 1 (D) 0

Q.74 (sin 43º cos 47º + cos 43º cos 47º) = ?
(A) 0 (B) 1
(C) sin 4º (D) cos 4º

Q.75 $\left(\dfrac{\sin 49^\circ}{\cos 41^\circ} - \dfrac{\cos 17^\circ}{\sin 73^\circ}\right) = ?$
(A) 1
(B) 0
(C) –1
(D) Cannot be calculated

Q.76 (sin 79º cos 11º + cos 79º sin 11º) = ?
(A) $\dfrac{1}{\sqrt{2}}$ (B) $\dfrac{1}{2}$
(C) 0 (D) 1

Q.77 $\dfrac{\cot(90^\circ-\theta)\cdot\sin(90^\circ-\theta)}{\sin\theta} + \dfrac{\cot 40^\circ}{\tan 50^\circ} - (\cos^2 20^\circ + \cos^2 70^\circ) = ?$
(A) 3 (B) 2
(C) 1 (D) 0

Q.78 $\dfrac{2\tan^2 30^\circ \sec^2 52^\circ \sin^2 38^\circ}{(\mathrm{cosec}^2 70^\circ - \tan^2 20^\circ)} = ?$
(A) $\dfrac{3}{2}$ (B) $\dfrac{2}{3}$
(C) 2 (D) $\dfrac{1}{2}$

Q.79 $\left\{\dfrac{(\sin^2 22^\circ+\sin^2 68^\circ)}{(\cos^2 22^\circ+\cos^2 68^\circ)} + \sin^2 63^\circ + \cos 63^\circ \sin 27^\circ\right\} = ?$
(A) 0 (B) 1
(C) 2 (D) 3

Q.80 (cosec^2 57º – $\tan^2$ 33º) = ?
(A) 0 (B) 1
(C) 2 (D) None of these

Q.81 ($\cos^2$ 28º – $\sin^2$ 62º) = ?
(A) 0 (B) 1
(C) 2 (D) None of these

Q.82 ($\sec^2$ 10º – $\cot^2$ 80º) = ?
(A) 1 (B) 0
(C) $\dfrac{3}{2}$ (D) None of these

Q.83 cos (40º + θ) – sin (50º – θ) = ?
(A) 1 (B) 0
(C) sin 2θ (D) None of these

Q.84 sin (45º + θ) – cos (45º – θ) = ?
(A) 2 cos θ (B) 2 sin θ
(C) 0 (D) 1

Q.85 cosec (75º + θ) – sec (15º – θ) = ?
(A) 2 sec θ (B) 2 cosec θ
(C) 0 (D) 1

Q.86 If sin (θ + 34º) = cos θ and θ is acute, then θ = ?
(A) 56º (B) 66º
(C) 28º (D) 42º

Q.87 $\sin\theta\cos(90^\circ-\theta)+\cos\theta\sin(90^\circ-\theta)=?$

(A) 0 (B) 1

(C) 2 (D) $\frac{3}{2}$

Q.88 $\dfrac{\cos 38^\circ \operatorname{cosec} 52^\circ}{\tan 18^\circ \tan 35^\circ \tan 60^\circ \tan 72^\circ \tan 55^\circ}=?$

(A) $\sqrt{3}$ (B) $\frac{1}{3}$

(C) $\frac{1}{\sqrt{3}}$ (D) $\frac{2}{\sqrt{3}}$

Q.89 If $\sin 3A=\cos(A-10^\circ)$ where 3A is an acute angle, then $\angle A=?$

(A) 35° (B) 25°

(C) 20° (D) 45°

Q.90 If $\tan 2A=\cot(A-21^\circ)$, where 2A is an acute angle, then $\angle A=?$

(A) 24° (B) 27°

(C) 35° (D) 37°

Q.91 If $\sec 5A=\operatorname{cosec}(A-30^\circ)$, where 5A is an acute angle, then $\angle A=?$

(A) 35° (B) 25°

(C) 20° (D) 27°

Q.92 If A and B are acute angles such that $\sin A=\cos B$, then $(A+B)=?$

(A) 45° (B) 60°

(C) 90° (D) 180°

Q.93 If $\cos A+\cos^2 A=2$, then the value of $\sin^2 A+\sin^4 A$ is **[NTSE 2012]**

(A) 1 (B) $\frac{1}{2}$

(C) 2 (D) 3

Q.94 The angles of elevations of the top of the tower from two points in the same straight line and at a distance of 9 m and 16 m from the base of the tower are complementary. The height of the tower is **[NTSE 2012]**

(A) 18 m (B) 16 m

(C) 10 m (D) 12 m

Q.95 The value of $\tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots\ldots \tan 89^\circ$ is **[NTSE 2013]**

(A) 0 (B) 1

(C) 2 (D) 3

Q.96 I left home at 3:00 pm and returned at 3:48 pm. The clock was rotated by 45°, so that when I left, the hour hand of a clock was pointing along the South-East direction. In which direction would the hour hand point when I returned? **[NTSE 2013]**

(A) 15° East of South

(B) 21° East of South

(C) 63° South of East

(D) 27° South of East

ANSWER KEY

1.	B	**2.**	A	**3.**	D	**4.**	A
5.	A	**6.**	C	**7.**	C	**8.**	A
9.	B	**10.**	D	**11.**	D	**12.**	C
13.	A	**14.**	C	**15.**	D	**16.**	B
17.	C	**18.**	C	**19.**	D	**20.**	B
21.	C	**22.**	B	**23.**	D	**24.**	C
25.	A	**26.**	A	**27.**	A	**28.**	A
29.	B	**30.**	A	**31.**	B	**32.**	C
33.	D	**34.**	B	**35.**	B	**36.**	C
37.	D	**38.**	B	**39.**	A	**40.**	B
41.	D	**42.**	A	**43.**	D	**44.**	D
45.	A	**46.**	D	**47.**	C	**48.**	B
49.	C	**50.**	A	**51.**	D	**52.**	B
53.	A	**54.**	C	**55.**	D	**56.**	C
57.	B	**58.**	A	**59.**	B	**60.**	C
61.	A	**62.**	B	**63.**	B	**64.**	D
65.	C	**66.**	B	**67.**	D	**68.**	C
69.	D	**70.**	D	**71.**	B	**72.**	C
73.	D	**74.**	B	**75.**	B	**76.**	D
77.	C	**78.**	B	**79.**	C	**80.**	B
81.	A	**82.**	A	**83.**	B	**84.**	C
85.	C	**86.**	C	**87.**	B	**88.**	C
89.	B	**90.**	D	**91.**	C	**92.**	C
93.	A	**94.**	D	**95.**	B	**96.**	B

SOME APPLICATIONS OF TRIGONOMETRY

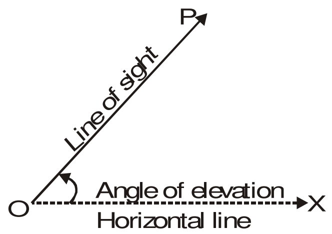
APPLICATIONS OF TRIGONOMETRY

Many times, we have to find the height and distances of many objects in real life. We use trigonometry to solve problems, such as finding the height of a tower, height of a flag mast, distance between two objects, where measuring directly is trouble, some and some times impossible. In those cases, we adopt indirect methods which involves solution of right triangles.

Thus Trigonometry is very useful in geography, astronomy and navigation. It helps us to prepare maps, determine the position of a landmass in relation to the longitudes and latitudes. Surveyors have made use of this knowledge since ages.

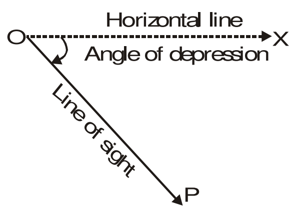
ANGLE OF ELEVATION

The angle between the horizontal line drawn through the observer eye and line joining the eye to any object is called the angle of elevation of the object, if the object is at a higher level than the eye i.e., If a horizontal line OX is drawn through O, the eye of the observer, and P is an object in the vertical plane through OX, then if P is above OX, as in fig. $\angle XOP$ is called the angle of elevation or the altitude of P as seen from O.



ANGLE OF DEPRESSION

The angle between the horizontal line drawn through the observer eye and the line joining the eye to any object is called the angle of depression of the object if the object is at a lower level than the eye. i.e., If a horizontal line OX is drawn through O, the eye of an observer, and P is an object in the vertical plane through OX, then if P is below OX, as in fig. $\angle XOP$ is called the angle of depression of P as seen from O.



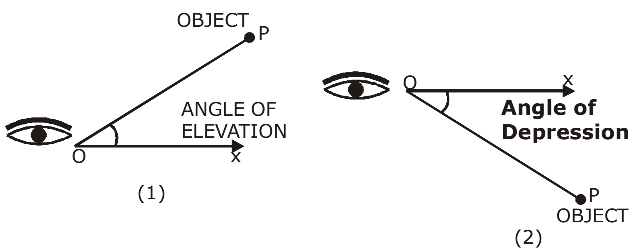
IMPORTANT DEFINITIONS

* **Angle of elevation and angle of depression**

Let O be an eye of an observer and OX be the horizontal line drawn through O. Let P be an object, then

(i) if P is above OX as in figure (1), $\angle XOP$ is called **angle of elevation** (as observed from O).

(ii) if P is below OX as in figure (2), $\angle XOP$ is called **angle of depression** (as observed from O).



The line OP, joining the eye to the object, is called the **line of sight.**

SOLVED PROBLEMS

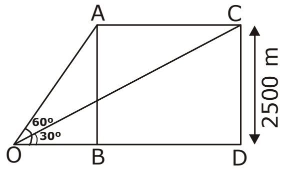
Ex.1 An aeroplane flying horizontally at a height of 2500 m. above the ground is observed at an elevation of 60º, and after 15 seconds, the elevation is observed to be 30º. Find the speed of the aeroplane in km/hr.

Sol. Initially the aeroplane is at A and after 15 seconds its position is at C.

$\therefore$ AC is distance covered in 15 sec.

Also $\angle AOB = 60^\circ ; \angle COD = 30^\circ$

AB = CD = 2500 m



In rt. ΔABO.

$$\frac{OB}{AB} = \cot 60^\circ \Rightarrow OB = \frac{2500}{\sqrt{3}} \qquad ..(i)$$

In rt. ΔCDO.

$$\frac{OD}{CD} = \cot 30^\circ \Rightarrow OD = 2500\sqrt{3} \qquad ..(ii)$$

From (i) and (ii),

$$BD = OD - OB = 2500\sqrt{3} - \frac{2500}{\sqrt{3}} = 2500\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) BD = 2500\left(\frac{3-1}{\sqrt{3}}\right)$$

$$= \frac{2500 \times 2 \times \sqrt{3}}{3} = \frac{5000 \times 1.732}{3} = \frac{8660}{3} \text{ m.}$$

$\frac{8660}{3}$ m is the distance covered in 15 sec.

$\therefore$ Speed

$$= \frac{8660}{3 \times 15} \text{m/sec} = \frac{8660}{3 \times 15 \times 1000} \times 60 \times 60 \text{ km/hr} = 692.8 \text{ km/hr.}$$

$\therefore$ Speed of the aeroplane is 692.8 km/hr.

Ex.2 The angle of elevation of the top of a tower from two points distant a and b from the base and in the same straight line with it are complementary. Prove that the height of the tower is $\sqrt{ab}$. [Foreign 2005, Delhi 2005 C]

Sol. Let OT be the tower of height h.

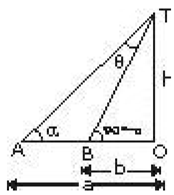
Let O be the base of the tower.

Let A and B be two points on the same line through the base such that OA = a and OB = b.

Because the angles at A and B are complementary, so let

$\angle TAO = \alpha$ and therefore

$\angle TBO = 90^\circ - \alpha$.



In ΔAOT. ..(i)

$$\tan \alpha = \frac{OT}{OA} = \frac{h}{a}$$

In ΔBOT

$$\tan (90^\circ - \alpha) = \frac{OT}{OB} = \frac{h}{b} \text{ or } \cot \alpha = \frac{h}{b} \qquad ..(ii)$$

Multiplying (i) and (ii), we have,

$\tan\alpha \cot\alpha = \frac{h}{b}\times\frac{h}{a} \Rightarrow \tan\alpha.\frac{1}{\tan\alpha} = \frac{h^2}{ab}$

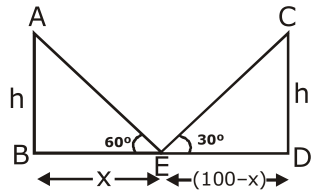
$\Rightarrow 1 = \frac{h^2}{ab} \Rightarrow h^2 = ab \Rightarrow h = \sqrt{ab}$.

Ex.3 Two pillars of equal height are on either side of a road, which is 100 m wide. The angles of evlevation of the top of the pillars are 60º and 30º at a point on the road between the pillars. Find the position of the point between the pillars. Also find the height of each pillar.

Sol. Let AB and CD be two pillars of equal height h m.

Let E be a point on the road such that

BE = x m. DE = (100 – x) m, $\angle AEB = 60^\circ$, $\angle CED = 30^\circ$.



In rt. triangle ABE,

$\frac{AB}{BE} = \tan 60^\circ \Rightarrow \frac{h}{x} = \sqrt{3} \Rightarrow h = \sqrt{3}x$ ..(i)

In rt. triangle CDE.

$\frac{CD}{DE} = \tan 30^\circ \qquad \frac{h}{100-x} = \frac{1}{\sqrt{3}} \Rightarrow h = \frac{100-x}{\sqrt{3}}$ ..(ii)

From (i) and (ii), we get

$\sqrt{3}x = \frac{100-x}{\sqrt{3}} \qquad \Rightarrow 3x = 100 - x$

$\Rightarrow 4x = 100 \qquad \Rightarrow \qquad x = 25$.

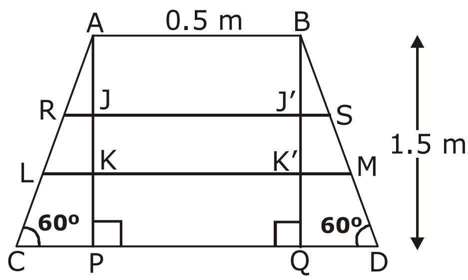
Putting x = 25 in (i), we get

$h = \sqrt{3} \times 25 = 25 \times 1.732 = 43.3$ m.

$\therefore$ Height of each pillar = 43.3 m; position of the point from a pillar making an angle of 60º is 25 m.

Ex.4 In the given figure, ABDC is a trapezium in which AB || CD. Line segment RS and LM are drawn parallel to AB such that AJ = JK = KP. If AB = 0.5 m and AP = BQ = 1.5 m. Find the length of AC, BD, RS and LM.

Sol. AB||RS||LM||CD.



$\therefore \qquad \angle ARJ = \angle ALK = \angle ACP = 60^\circ$

AJ = JK = KP = 0.5 m.

In rt. ΔAPC,

$\frac{AC}{AP} = \text{cosec } 60^\circ \Rightarrow AC = 1.5 \times \frac{2}{\sqrt{3}}$

$= \frac{15}{10}\times\frac{2\times\sqrt{3}}{3} = \sqrt{3} = 1.73$ m AC = BD

Now, RS = RJ + JJ′ + J′S = 2RJ + 0.5 ...(i)

In rt. ΔAJR [$\because$ RJ = J′S]

$\frac{RJ}{AJ} = \cot 60^o \Rightarrow RJ = 0.5 \times \frac{1}{\sqrt{3}} = \frac{5}{10} \times \frac{\sqrt{3}}{3}$

$\therefore$ RJ = 0.288 m

Substituting in (i), we get

RS = 0.5 + 2 × 0.288 = 0.5 + 0.576 = 1.076 m.

Now LM = LK + KK′ + K′M = 2LK + 0.5 ...(ii)

[$\because$ LK = K′M]

In rt. ΔAKL, $\frac{LK}{AK} = \cot 60^o \Rightarrow LK = 0.1 \times \frac{1}{\sqrt{3}} = 0.577$ m

Substituting in (ii), we get

LM = 2 × 0.577 + 0.5

= 1.3856 + 0.5 = 1.654 m.

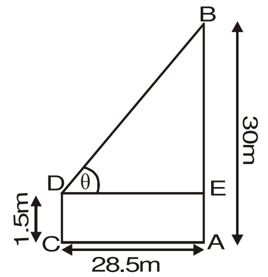
Ex.5 An observer 1.5 m tall, is 28.5 m away from a tower 30 m high. Determine the angle of elevation of the top of the tower from his eye.

Sol. Let AB be the height of the tower, CD the height of the observer with his eye at the point D, AB = 30 m, CD = 1.5 m.

Through D, draw DE II CA then $\angle$BDE = θ where θ is the angle of elevation of the top of the tower from his eye. AC = horizontal distance between the tower and the observer = 28.5 m

BE = AB - AE = (30 - 1.5) m = 28.5 m BDE is right triangle at E,

then $\frac{BE}{DE} = \tan\theta \Rightarrow \frac{28.5}{28.5} = \tan\theta \Rightarrow \tan\theta = 1$



$\Rightarrow \tan\theta = 1 = \tan 45° \Rightarrow \theta = 45°$.

Required angle of elevation of the tower = θ = 45°.

Ex.6. A vertical post casts a shadow 21 m long when the altitude of the sun is 30°. Find :

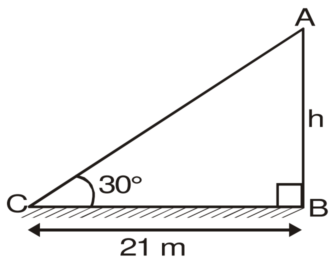
(a) the height of the post.

(b) the length of the shadow when the altitude of the sun is 60°.

(c) the altitude of the sun when the length of the shadow is $7\sqrt{3}$ m.

Sol. Let AB be the vertical post and its shadow is 21 m when the altitude of the sun is 30°.

(a) BC = 21 m, $\angle$ACB = 30°, AB = h metres ABC is rt. Δ, $\frac{AB}{BC} = \tan 30°$



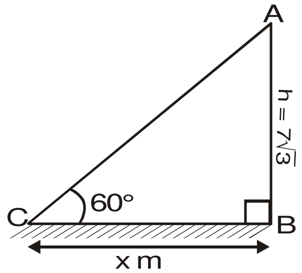
$\Rightarrow \frac{h}{21} = \frac{1}{\sqrt{3}}$

$\Rightarrow h = \frac{21}{\sqrt{3}} = \frac{7 \times \sqrt{3} \times \sqrt{3}}{\sqrt{3}} = 7\sqrt{3}$ m

$\Rightarrow$ AB = h, Height of the pole = $7\sqrt{3}$ m

(b) In this case, we have,

$\angle$ACB = 60°, BC = x m, AB = $7\sqrt{3}$ m ABC is rt. Δ, then :



$\frac{AB}{BC} = \tan 60° = \sqrt{3}$

$\Rightarrow \quad \frac{h}{x} = \sqrt{3} \Rightarrow \frac{7\sqrt{3}}{x} = \sqrt{3}$

$\Rightarrow$ x = BC, Length of the shadow = 7 m.

(c) In this case:

AB = h = $7\sqrt{3}$ m

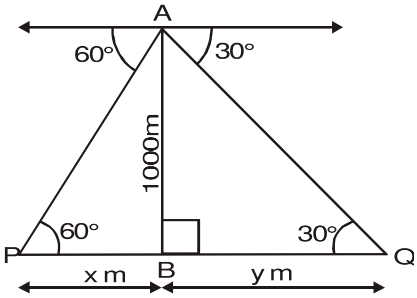
BC = The length of the shadow = $7\sqrt{3}$ m

when the altitude of the sun is θ

ABC is rt. Δ, then $\frac{AB}{BC} = \tan\theta = \frac{7\sqrt{3}}{7\sqrt{3}} = \tan\theta$ $\tan\theta = 1 = \tan 45° \Rightarrow \theta = 45°$

Altitude of the sun = θ = 45°

Ex.7 A captain of an aeroplane flying at an altitude of 1000 metres sights two ships as shown in the figure. If the angle of depressions are 60° and 30°, find the distance between the ships.



Sol. Let A be the position of the captain of an aeroplane flying at the altitude of 1000 metres from the ground.

AB = the altitude of the aerplane from the ground = 1000 m

P and Q be the position of two ships.

Let PB = x metres, and BQ = y metres.

Required : PQ = Distance between the ships = (x + y) metres.

ABP is rt. Δ at B

$\frac{AB}{PB} = \tan 60°$

$\frac{1000}{x} = \sqrt{3} \Rightarrow x = \frac{1000}{\sqrt{3}}$

ABQ is rt. Δ at B

$\frac{AB}{BQ} = \tan 30°$

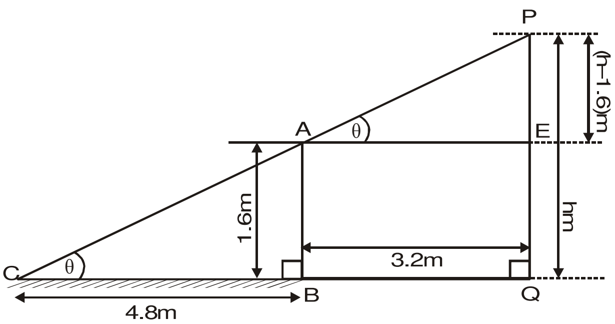
$\frac{1000}{y} = \frac{1}{\sqrt{3}} \Rightarrow y = 1000\sqrt{3}$

$x = \frac{1000\,(1.732)}{3} = 577.3$ m $\quad y = 1000\,(1.732) = 1732$ m

Required distance between the ships = (x+y) metres = (577.3+1732) m = 2309.3 m

Ex.8 A 1.6 m tall girl stands at a distance of 3.2 m from the lamp-post and casts a shadow of 4.8 m on the ground. Find the height of the lamp-post by using (i) trigonometric ratios (ii) property of similar triangles.

Sol. Let PQ be the position of the lamp-post whose height is h metres. i.e., PQ = h metres. AB be the position of the tall girl such that AB = 1.6 m. Let BC be the shadow of AB such that BC = 4.8 m, $\angle ACB = \angle PAE = \theta$ (corr. $\angle$s)



(i) In right Δ ABC, $\frac{AB}{BC} = \tan\theta \Rightarrow \frac{1.6}{4.8} = \tan\theta \Rightarrow \tan\theta = \frac{1}{3}$

AB = EQ = 1.6 m. Also AE = BQ = 3.2 m

PE = PQ - EQ = (h - 1.6) m

In right Δ AEP, $\frac{PE}{AE} = \tan\theta \Rightarrow \frac{(h-1.6)}{3.2} = \tan\theta = \frac{1}{3} \Rightarrow \frac{h-1.6}{3.2} = \frac{1}{3}$

$3h - 4.8 = 3.2 \Rightarrow 3h = 4.8 + 3.2 = 8 \Rightarrow 3h = 8$

The height of the lamp-post = $\frac{8}{3}$ m = $2\frac{2}{3}$ m

(ii) In two Δs ACB and PCQ, we have:

$\angle ACB = \angle PCQ = \theta$ (common)

$\angle ABC = \angle PQC = 90° \Rightarrow \Delta ACB \sim \Delta PAE$

CQ = CB + BQ

= (4.8 + 3.2) m = 8 metres

(AA similarity)

$\frac{AC}{PC} = \frac{CB}{CQ} = \frac{AB}{PQ} \Rightarrow \frac{BC}{CQ} = \frac{AB}{PQ}$

$\frac{BC}{CQ} = \frac{AB}{PQ} \Rightarrow \frac{4.8}{8} = \frac{1.6}{h} \Rightarrow \frac{3}{8} = \frac{1}{h} \Rightarrow 3h = 8$

Thus $3h = 8 \Rightarrow h = \frac{8}{3}$ m $\Rightarrow h = 2\frac{2}{3}$ m

Required height of the lamp-post = PQ = h = $2\frac{2}{3}$ m

EXERCISE – I

UNSOLVED PROBLEMS

Q.1 From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45º and 60º respectively. Find the height of the tower.

Q.2 From a point P on the ground the angle of elevation of the top of a 10 m tall building is 30º. A flag is hoisted at the top of the building and the angle of elevation of the top of the flagstaff from P is 45º. Find the length of the flagstaff and the distance of the building from the point P. (Take $\sqrt{3}$ = 1.732)

Q.3 A statue, 1.6 m tall, stands on the top of pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60º and from the same point the angle of elevation of the top of the pedestal is 45º. Find the height of the pedestal.

Q.4 From a point P on the ground, the angle of elevation of the top of a 10 m tall building and a helicopter, hovering over the top of the building, are 30º and 60º respectively. Find the height of the helicopter above the ground.

Q.5 A window in a building is at a height of 10 m from the ground. The angle of depression of a point P on the ground from the window is 30º. The angle of elevation of the top of the building from the point P is 60º. Find the height of the building.

Q.6 The angle of elevation of the top of a building from the foot of a tower is 30º and the angle of elevation of the top of the tower from the foot of the building is 60º. If the tower is 50 m high, find the height of the building.

Q.7 A T.V. tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60º. From another point 20 m away from this point on the line joining this point to the foot of tower, the angle of elevation of the top of the tower is 30º. Find the height of the tower and the width of the canal.

Q.8 The angle of elevation of the top of a tower from a point on the same level as the foot of the tower is 30º. On advancing 150 metres towards the foot of the tower, the angle of elevation becomes 60º. Find the height of the tower.

Q.9 The shadow of a tower standing on a level ground is found to be 40 m longer when the sun's altitude is 30º than when it is 60º. Find the height of the tower.

Q.10 As observed from the top of a 75 m high light house from the sea-level, the angles of depression of two ships are 30º and 45º. If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships.

Q.11 From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30º and 45º respectively. If the bridge is at a height of 3 m from the bank, find the width of the river.

Q.12 From the top of a cliff 150 m high, the angles of depression of two boats are 60º and 30º. Find the distance between the boats, if the boats are

(i) on the same side of the cliff.

(ii) on the opposite sides of the cliff.

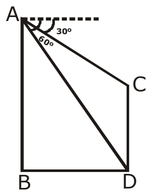
Q.13 Two pillars of equal height stand on either side of a roadway which is 80 m wide. At a point on the road between pillars, the elevations of the pillars are 60º and 30º. Find the height of the pillars and the position of the point.

Q.14 There is a small island in between a river 100 metres wide. A tall tree stands on the island. P and Q are points directly opposite each other on the two banks, and in line with the tree. If the angles of elevation of the top of the tree from P and Q are 30º and 45º respectively, find the height of the tree.

Q.15 (i) The angles of elevation of the top of a tower from two points P and Q at distance of a and b respectively from the base and in the same straight line with it are complementary. Prove that the height of the tower is $\sqrt{ab}$

(ii) The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

Q.16 In the adjoining figure from the top of a building AB, 60 metres high, the angles of depression of the top and the bottom of a vertical lamp post CD are observed to be 30º and 60º respectively. Find



(i) the horizontal distance AB and CD.

(ii) the height of the lamp post CD.

Q.17 The angles of depression of the top and the bottom of an 8 m tall building from the top of a multi-storeyed building are 30º and 45º respectively. Find the height of the multi-storeyed building and the distance between the two buildings.

Q.18 From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60º and the angle of depression of its foot is 45º. Determine the height of the tower.

Q.19 From the base of a 30 m high building, the angle of elevation of a tower is 60º, and from the top of the building, it is 30º. Find the height of the tower and distance between the building and the tower.

Q.20 A man standing on the deck of a ship, which is 10 m above water level, observes the angle of elevation of the top of a hill as 60º and angle of depression of the base of the hill as 30º.Find the distance of the hill from the ship and height of the hill.

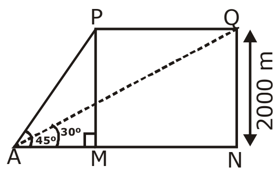
Q.21 The height of a house subtends a right angle at the opposite window. The angle of elevation angle of the window from the base of the house is 60º. If the width of the road is 6 m, find the height of the house.

Q.22 (i) The angle of elevation of a bird from a point 50 metres above a lake is 30º and the angle of depression of its reflection in the lake is 60º. Find the the height of the bird above the lake.

(ii) The angle of depression of a cloud from a point 200 metres above a lake is 30º and the angle of depression of its reflection in the lake is 60º. Find the height of the cloud.

Q.23 A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eye to the top of the building increases from 30º to 60º as he walks towards the building. Find the distance he walked towards the building.

Q.24 In the adjoining figure, the angle of elevation of a helicopter from a point A on the ground is 45º. After 15 seconds flight, the angle of elevation changes to 30º. If the helicopter is flying at a height of 2000 m, find the speed of the helicopter. (Take $\sqrt{3}$ = 1.732)



ANSWER KEY

1. $20(\sqrt{3}-1)$ m **2.** 7.32 m; 17.32 m

3. $0.8(\sqrt{3}+1)$ m **4.** 30 m **5.** 30 m

6. $16\frac{2}{3}$ m **7.** $10\sqrt{3}$ m ; 10 m

8. 75 $\sqrt{3}$ m **9.** 20 $\sqrt{3}$ m **10.** 75 $(\sqrt{3}-1)$ m

11. $3\left(\sqrt{3}+1\right)$ m **12.** (i) 100 $\sqrt{3}$ m (ii) $200\sqrt{3}$ m

13. $20\sqrt{3}$ m, 20 m from the pillar whose angle of elevation is 60º **14.** $50(\sqrt{3}-1)$ m

16. (i) $20\sqrt{3}$ m (ii) 40 m

17. $4\left(3+\sqrt{3}\right)$ m; $4\left(3+\sqrt{3}\right)$ m **18.** $7\left(\sqrt{3}+1\right)$ m

19. 45 m ; $15\sqrt{3}$ m **20.** $10\sqrt{3}$ m ; 40 m

21. $8\sqrt{3}$ m **22.** (i) 100 m (ii) 400 m

23. $19\sqrt{3}$ m **24.** 97.6 m/sec.

EXERCISE – II

BOARD PROBLEMS

Q.1 On a horizontal plane there is a vertical tower with a flag pole on the top of the tower. At a point 9 metres away from the foot of the tower the angle of elevation of the top and bottom of the flag pole are 60° and 30° respectively. Find the height of the tower and flag pole mounted on it.

OR

From a building 60 metres high the angles of depression of the top and bottom of lamp-post are 30° and 60° respectively. Find the distance between lamp-post and building. Also find the difference of height between building and lamp-post. **[Delhi-2005]**

Q.2 From the top of a cliff 92 cm high, the angle of depression of a boy is 20°. Calculate to the nearest metre, the distance of the boy from the foot of the cliff. **[ICSE-2005]**

Q.3 The shadow of a vertical tower AB on level ground is increased by 10 m, when the altitude of the sun changes from 45° to 30°. Find the height of the tower and give your answer correct to $\frac{1}{10}$ of a metre. **[ICSE-2006]**

Q.4 The angles of depression of the top and the bottom of a building 50 metres high as observed from the top of a tower are 30° and 60° respectively. Find the height of the tower and also the horizontal distance between the building and the tower. **[Delhi-2006]**

OR

The angle of elevation of the top of a tower as observed from a point on the ground is 'α' and on moving 'a' metres towards the tower, the angle of elevation is 'β'. Prove that the height of the tower is $\frac{a \tan\alpha \tan\beta}{\tan\beta - \tan\alpha}$.

Q.5 A man on the top of a vertical tower observes a car moving at a uniform speed coming directly towards it. If it takes 12 minutes for the angle of depression to change from 30° to 45° how soon after this, will the car reach the tower? **[AI-2006C]**

Q.6 A boy standing on a horizontal plane finds a bird flying at a distance of 100 m from him at an elevation of 30°. A girl standing on the roof of 20 metre high building, finds the angle of elevation of the same bird to be 45°. Both the boy and the girl are on opposite sides of the bird. Find the distance of bird from the girl. **[Delhi-2007]**

Q.7 Statue 1.46 m tall, stands on the top of the pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point, the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal. (use $\sqrt{3}$ = 1.73) **[CBSE-Delhi-2008]**

Q.8 A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on the opposite bank is 60°. When he moves 40 m away from the bank, he finds the angle of elevation to be 30°. Find the height of the tree and the width of the river. (use $\sqrt{3}$ = 1.732) **[CBSE-Delhi-2008]**

Q.9 The angle of elevation of a jet fighter from a point A on the ground is 60°. After a fight of 15 seconds, the angle of elevation changes to 30°. If the jet is flying at a speed of 720 km/hour, find the constant height at which the jet is flying. (use $\sqrt{3}$ = 1.732) **[CBSE-AI-2008]**

Q.10 The angle of elevation of an aeroplane from a point A on the ground is 60°. After a flight of 30 seconds, the angle of elevation changes to 30°. If the plane is flying at a constant height of 3600 $\sqrt{3}$ m, find the speed in km/hour of the plane. **[CBSE-foreign-2008]**

Q.11 A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point. **[CBSE-Delhi-2009]**

Q.12 An aeroplane when flying at a height of 3125 m from the ground passes vertically below another plane at an instant when the angles of elevation of the two planes from the same point on the ground are 30° and 60° respectively. Find the distance between the planes at that instant **[CBSE-AI-2009]**

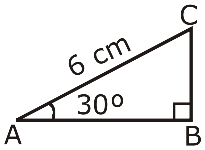
Q.13 A man is standing on the deck of a ship which is 25m above water level. He observes the angle of elevation of the top of a lighthouse as 60° and the angle of depression of the base of the light house as 45°. Calculate the height of the lighthouse.

ANSWER KEY

1. 15.588 m and 5.196 m or 34.64 m and 20 m
2. 253 m **3.** 13.66 m **4.** 75 m and 43.3 m
5. 16 minutes 23 seconds **6.** $30\sqrt{2}$
7. 2 m **8.** 34.64 m and 20 m **9.** 2598 m
10. 864 km/hr **11.** 3 seconds
12. 2083.33 m **13.** 68.25 m

EXERCISE – III | MULTIPLE CHOICE QUESTIONS

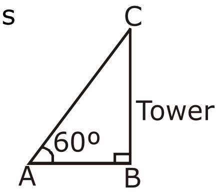
Q.1 In the adjoining figure, the length of BC is



(A) $2\sqrt{3}$ cm (B) $3\sqrt{3}$ cm

(C) $4\sqrt{3}$ cm (D) 3 cm

Q.2 In the adjoining figure, if the angle of elevation is 60° and the distance AB = $10\sqrt{3}$ m, then the height of the tower is



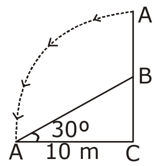
(A) $20\sqrt{3}$ cm
(B) 10 m
(C) 30 m
(D) $30\sqrt{3}$ m

Q.3 If a kite is flying at a height of $40\sqrt{3}$ metres from the level ground, attached to a string inclined at 60° to the horizontal, then the length of the string is

(A) 80 m (B) $60\sqrt{3}$ m

(C) $80\sqrt{3}$ m (D) 120 m

Q.4 The top of a broken tree has its top touching the ground (shown in the adjoining figure,) at distance of 10 m from the bottom. If the angle made by the broken part with ground is 30°, then the length of the broken part is



(A) $10\sqrt{3}$ m
(B) $\frac{20}{\sqrt{3}}$ m
(C) 20 m
(D) $20\sqrt{3}$ m

Q.5 If the angle of depression of an object from a 75 m high tower is 30°, then the distance of the object from the tower is

(A) $25\sqrt{3}$ m (B) $50\sqrt{3}$ m

(C) $75\sqrt{3}$ m (D) 150 m

Q.6 A ladder 14 m long rests against a wall. If the foot of the ladder is 7 m from the wall, then the angle of elevation is

(A) 15° (B) 30°
(C) 45° (D) 60°

Q.7 If the length of the shadow of a tower is $\sqrt{3}$ times that of its height, then the angle of elevation of the sun is

(A) 15° (B) 30°
(C) 45° (D) 60°

Q.8 In ΔABC, $\angle A = 30°$ and $\angle B = 90°$. If AC = 6 cm, then its area is

(A) $16\sqrt{3}$ cm^2 (B) 16 cm^2

(C) $8\sqrt{3}$ cm^2 (D) $6\sqrt{3}$ cm^2

Q.9 At a certain instant the ratio of the lengths of a pillar and its shadow are in the ratio $1:\sqrt{3}$. At the that instant, the angle of elevation of the sun is

(A) 30° (B) 45°
(C) 60° (D) none of these

Q.10 At a certain instant, the altitude of the sun is 60°. At that instant, the length of the shadow of a vertical tower is 100m. The height of the tower is

(A) $50\sqrt{3}$ m (B) $100\sqrt{3}$ m

(C) $\frac{100}{\sqrt{3}}$ m (D) $\frac{200}{\sqrt{3}}$ m

Q.11 The height of a tower is $100\sqrt{3}$ m. The angle of elevation of its top from a point 100 m away from its foot is

(A) 30° (B) 45°
(C) 60° (D) none of these

Q.12 The angle of elevation of the top of a tower from a point on the ground 30 m away from the foot of the tower is 30°. The height of the tower is

(A) 30 m (B) $10\sqrt{3}$ m

(C) 20 m (D) $10\sqrt{2}$ m

Q.13 The string of a kite is 100 m long and it makes an angle of 60° with the horizontal. If there is no slack in the string, the height of the kite from the ground is

(A) $50\sqrt{3}$ m (B) $100\sqrt{3}$ m

(C) $50\sqrt{2}$ m (D) 100 m

Q.14 An observer 12.5 m tall is 28.5 m away from tower and the angle of elevation of the top of the tower from the eye of the observer is 45°. The height of the tower is

(A) 27 m (B) 30 m
(C) 28.5 m (D) none of these

Q.15 The shadow of a vertical tower on level ground increases by 10 m when the altitude of the sun changes from 45° to 30°. The height of the tower is

(A) $5(\sqrt{3}+1)$ m (B) $10(\sqrt{3}-1)$ m

(C) 9 m (D) 13 m

Q.16 From the top of a hill, the angles of depression of two consecutive km stones due east are found to be 30º and 45º. The height of the hill is

(A) $\frac{1}{2}\left(\sqrt{3}-1\right)$ km (B) $\frac{1}{2}\left(\sqrt{3}+1\right)$ km

(C) $\left(\sqrt{3}-1\right)$ km (D) $\left(\sqrt{3}+1\right)$ km

Q.17 An aeroplane at an altitude of 200 m obseves the angles of depression of opposite points on the two banks of a river to be 45º and 60º. The width of the river is

(A) $\left(200+\frac{200}{\sqrt{3}}\right)$ m (B) $\left(200-\frac{200}{\sqrt{3}}\right)$ m

(C) $400\sqrt{3}$ m (D) $\frac{400}{\sqrt{3}}$ m

Q.18 If the angles of elevation of the top of a tower from two points at distances a and b from the base and in the same straight line with it are complementary, then the height of the tower is

(A) $\sqrt{\frac{a}{b}}$ (B) $\sqrt{ab}$

(C) $\sqrt{a+b}$ (D) $\sqrt{a-b}$

Q.19 On the level ground the angle of elevation of a tower is 30º. On moving 20 m nearer, the angle of elevation is 60º. The height of the tower is

(A) 10 m (B) $10\sqrt{3}$ m

(C) 15 m (D) 20 m

Q.20 If the elevation of the sun changes from 30º to 60º, then the difference between the length of shadows of a pole 15 m high, is

(A) 7.5 m (B) 15 m

(C) $10\sqrt{3}$ m (D) $5\sqrt{3}$ m

Q.21 On the same side of tower 300 m high, the angles of depression of two objects are 45º and 60º respectively, then the distance between the object is (Take $\sqrt{3}=1.73$)

(A) 127 m (B) 117 m

(C) 217 m (D) 473 m

Q.22 The height s of two poles are 80 m and 65 m. If the line joining their tops makes an angle of 45º with the horizontal, then the distance between the poles is

(A) 15 m (B) 22.5 m

(C) 30 m (D) 7.5 m

Q.23 From the foot of a tower, the angle of elevation of the top of a column is 60º and from the top of the tower, the angle of elevation is 30º. If the height of the tower is 25 m, the height of the column is

(A) 35 m (B) 42.5 m

(C) 37.5 m (D) 27.5 m

Q.24 A straight tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle of 30º with the ground. The distance from the foot of the tree to the point, where the top touches the ground is 10 m. The height of the tree is

(A) $10\sqrt{3}$ m (B) $\frac{10\sqrt{3}}{3}$ m

(C) $10\left(\sqrt{3}+1\right)$m (D) $10\left(\sqrt{3}-1\right)$m

Q.25 An observer standing 50 m away from a building notices that the angles of elevation of the top and bottom of t flagstaff on the building are 60º and 45º respectively. The height of the flagstaff is

(A) $50\sqrt{3}$ m (B) $50\left(\sqrt{3}+1\right)$ m

(C) $50\left(\sqrt{3}-1\right)$ m (D) $\frac{50}{\sqrt{3}}$m

Q.26 A boat is being rowed away from a cliff 150 m high. At the top of the cliff the angle of depression of the boat changes from 60º to 45º in 1 minute. If $\sqrt{3}=1.73$, the speed of the boat is

(A) 4.31 km/hr (B) 3.81 km/hr

(C) 4.63 km/hr (D) 3.91 km/hr

Q.27 In a rectangle, the angle between a diagonal and a side is 30º and the length of this diagonal is 8 cm. The area of the rectangle is

(A) 16 cm^2 (B) $\frac{16}{\sqrt{3}}$cm^2

(C) $16\sqrt{3}$ cm^2 (D) $8\sqrt{3}$ cm^2

Q.28 If the length of shadow of a pole on a level ground is twice the length of the pole the angle of elevation of the sun is

(A) 30º (B) 45º

(C) 60º (D) none of these

Q.29 A pole of 6 m high casts a shadow $2\sqrt{3}$ m long on the ground. At that instant, the sun's elevation is

(A) 30º (B) 45º

(C) 60º (D) 90º

Q.30 Two men standing on opposite sides of a flagstaff measure the angles of the top of the flagstaff as 30º and 60º. If the height of the flagstaff is 18 m, the distance between the men is

(A) 24 m (B) $24\sqrt{3}$ m

(C) $\frac{24}{\sqrt{3}}$m (D) 31.2 m

Q.31 The angles of elevation of an aeroplane flying vertically above the ground as observed from two consecutive stones 1 km apart are 45° and 60°. The height of the aeroplane from the ground is.

(A) $\left(\sqrt{3}+1\right)$ km (B) $\left(3+\sqrt{3}\right)$ km

(C) $\frac{1}{2}\left(\sqrt{3}+1\right)$ km (D) $\frac{1}{2}\left(3+\sqrt{3}\right)$ km

Q.32 The angle of elevation of the sun, when the length of the shadow of a tree is $\sqrt{3}$ times the height of the tree, is: **[NTSE]**

(A) 30° (B) 45°
(C) 60° (D) 90°

Q.33 From a point P on a level ground, the angle of elevation of the top of a tower is 30°. If the tower is 100m high, the distance of point P from the foot of the tower is: **[NTSE]**

(A) 149m (B) 156m
(C) 173m (D) 200m

Q.34 The angle of elevation of a ladder leaning against a wall is 60° and the foot of the ladder is 4.6m away from the wall. The length of the ladder is: **[NTSE]**

(A) 2.3m (B) 4.6m
(C) 7.8m (D) 9.2m

Q.35 An observer 1.6m tall is $20\sqrt{3}$ m away from a tower. The angle of elevation from his eye to the top of the tower is 30°. The height of the tower is: **[NTSE]**

(A) 21.6m (B) 23.2m
(C) 24.72m (D) None of these

Q.36 Two ships are sailing in the sea on the two sides of a lighthouse. The angles of elevation of the top of the lighthouse as observed from the two ships are 30° and 45°, respectively. If the lighthouse is 100m high, the distance between the two ships is: **[NTSE]**

(A) 173m (B) 200m
(C) 273m (D) 300m

Q.37 A man standing at a point P is watching the top of a tower, which makes an angle of elevation of 30° with the man's eye. The man walks some distance towards the tower to watch its top the angle of elevation becomes 60°. What is the distance between the base of the tower and the point P? **[NTSE]**

(A) $4\sqrt{3}$ units (B) 8 units

(C) Data inadequate (D) None of these

Q.38 The angle of elevation of the top of a tower from a certain point is 30°. If the observer moves 20m towards the tower, the angle of elevation of the top of the tower increases by 15°. The height of the tower is: **[NTSE]**

(A) 17.3m (B) 21.9m
(C) 27.3m (D) 30m

Q.39 A man is watching from the top of a tower a boat speeding away from the tower. The boat makes an angle of depression of 45° with the man's eye when at a distance of 60 meters from the tower. After 5 seconds, the angle of depression becomes 30°. What is the approximate speed of the boat, assuming that it is running in still water? **[NTSE]**

(A) 32 kmph (B) 36 kmph
(C) 38 kmph (D) 40 kmph

Q.40 On the same side of a tower, two objects are located. Observed from the top of the tower, their angles of depression are 45° and 60°. If the height of the tower is 150m, the distance between the objects is: **[NTSE]**

(A) 63.5m (B) 76.9m
(C) 86.7m (D) 90m

Q.41 A man on the top of a vertical observation tower observes a car moving at a uniform speed coming directly towards it. If it takes 12 minutes for the angle of depression to change from 30° to 45°, how soon after this will the car reach the observation tower? **[NTSE]**

(A) 14 min. 35 sec. (B) 15 min. 49 sec.
(C) 16 min. 23 sec. (D) 18 min. 5 sec.

ANSWER KEY

1.	D	**2.**	C	**3.**	A	**4.**	B
5.	C	**6.**	D	**7.**	B	**8.**	C
9.	A	**10.**	B	**11.**	C	**12.**	B
13.	A	**14.**	B	**15.**	A	**16.**	B
17.	A	**18.**	B	**19.**	B	**20.**	C
21.	A	**22.**	A	**23.**	C	**24.**	A
25.	C	**26.**	B	**27.**	C	**28.**	D
29.	C	**30.**	B	**31.**	D	**32.**	A
33.	C	**34.**	D	**35.**	A	**36.**	C
37.	D	**38.**	C	**39.**	A	**40.**	A
41.	C						

EXERCISE – IV NTSE /OLYMPIAD /FOUNDATION PROBLEMS

Q.1 If angle of elevation of an aeroplane from two points at distance 1 metre apart are 60° and 30°, then the height of the aeroplane is-

[1] $\frac{500}{\sqrt{3}}$ m [2] $500\sqrt{3}$ m

[3] $\frac{2000}{\sqrt{3}}$ m [4] none of these

Q.2 A man is standing on a tower of height 300 m. He observes two cars on the same side whose angles of depression are 60° and 30°. The distance between these cars is-

[1] 25 m [2] 72 m

[3] 173.2 m [4] $200\sqrt{3}$ m

Q.3 The angle of elevation of a tower from a point A due south of it is 30° and from a point due west of it is 45°. If the heigth of the tower be 100 metres, then AB is equal to-

[1] 150 m [2] 200 m

[3] 173.2 m [4] 141.2 m

Q.4 A tower subtends α angle at a point A at its base plane. If the angle of depression of the foot of the tower at a point B at b height above A is β, then the height of the tower is-

[1] b tan α cot β [2] b cot α tan β

[3] b tan α tan β [4] b cot α cot β

Q.5 Angles of depression of the top of a hill of height 60 metres at the top and bottom of a pole on the ground are 30° and 60° respectively. The heigth of the pole is

[1] 20 m [2] 30 m

[3] 40 m [4] 50 m

Q.6 If a flag staff of 6 metres high placed on the top of a tower throws a shadow of $2\sqrt{3}$ metres along the ground, then the angle of elevation of the sun is-

[1] 75° [2] 60°

[3] 45° [4] 80°

Q.7 The angle of elevation of the top of a tower from a point A on the ground is 15°. If on walking 100 feet towards the tower, the angle of elevation becomes 30°, then the height of the tower is-

[1] 50 feet [2] $50\sqrt{3}$ feet

[3] $\frac{50}{\sqrt{3}}$ feet [4] none of these

Q.8 AB is a tower with its point A on the ground. C be its mid point and CB subtends α angle at a point P on the ground. If AP = nAB, then

[1] n = $(n^2 + 1)$ tan α

[2] n = $(2n^2 + 1)$ tan α

[3] n = $(2n^2 - 1)$ tan α

[4] n^2 = $(2n^2 + 1)$ tan α!

Q.9 AB is a vertical pole standing on the ground at A. Another point P on the ground is such that AP = 3AB. If C be the midpoint of AB and $\angle$CPB = β, then tan β is equal to-

[1] $\frac{1}{6}$ [2] $\frac{3}{19}$

[3] $\frac{18}{19}$ [4] none of these

Q.10 A balloon is observed simultaneously from three points A, B and C on a straigth road directly under it. The angular elevation at B is twice and at C is thrice that of A. If the distance between A and B is 200 metres and distance between B and C is 100 metres, then the heigth of the balloon is-

[1] 50 m [2] $50\sqrt{2}$ m

[3] $50\sqrt{3}$ m [4] none of these

Q.11 PQ is a vertical tower and P, Q are its bottom and top points respectively. A, B, C are three points on a horizontal line passing through P. Angles of elevation of Q from points A, B, C are equal to θ. If a, b, c are sides and Δ is the area of triangle ABC, then the height of the tower is-

[1] $\frac{abc}{4}$ [2] $\frac{abc \tan\theta}{4\Delta}$

[3] $\frac{4abc\ \Delta}{abc}$ [4] $\frac{4\tan\theta}{abc}$

Q.12 Among two towers T and R, the heigth of T is 72 metres which is less than that of R. If the angle of depression of the top of T from the top of R is

30° and its angle of elevation from the foot of R is 45, then the height of R is-

[1] $72\sqrt{3}$ m [2] $72(1+\sqrt{3})$ m

[3] $72(1+\frac{1}{\sqrt{3}})$ m [4] $72(1-\frac{1}{\sqrt{3}})$ m

Q.13 Angles of elevation of the top of a tower as observed from the bottom and top of a building of heigth 60 metres are 60° and 45° respectively. The distance of the base of the tower from the base of the building is-

[1] $30(\sqrt{3}-1)$ m [2] $30(\sqrt{3}+3)$ m

[3] $30(3-\sqrt{3})$ m [4] $30(\sqrt{3}+1)$ m

Q.14 The angle of elevation of the top of a temple situated on east side of a person is 60°. After walking 240 metres on north side it becomes 30°. The heigth of the temple is-

[1] $60\sqrt{6}$ m [2] 60 m

[3] $50\sqrt{3}$ m [4] $30\sqrt{6}$ m

Q.15 The angle of elevation of the top of a tower from a point 20 m away from its base is 45°. The height of the tower is-

[1] 10 m [2] 20 m

[3] 40 m [4] $20\sqrt{3}$ m

Q.16 From the top of a cliff 25 meters high, the angle of elevation of a tower is found to be equal to the angle of depression of the foot of the tower, then height of the tower is-

[1] 25 m [2] $25\sqrt{2}$ m

[3] $\frac{25}{\sqrt{2}}$m [4] 50 m

Q.17 Two poles of equal height stand on either side of a 100 meters wide road. At a point between the poles, the angle of elevation of the topes of the poles are 30° and 60°. The heigth of each pole is-

[1] 25 m [2] $25\sqrt{3}$ m

[3] $\frac{100}{\sqrt{3}}$m [4] none of these

Q.18 A person walking along a straight road observes that a two points 1 km apart, the angles of elevation of a pole in front of him are 30° and 75°. The heigth of the pole is-

[1] $250(\sqrt{3}+1)$ m [2] $250(\sqrt{3}-1)$ m

[3] $225(\sqrt{2}-1)$ m [4] $225(\sqrt{2}+1)$ m

Q.19 A tower subtends an angle α at a point A in the plane of its base and the angle of depression of the foot of the tower at a point ℓ metres just above A is β. The height of the tower is-

[1] $\ell \tan\beta \cot\alpha$ [2] $\ell \tan\alpha \cot\beta$

[3] $\ell \tan\alpha \tan\beta$ [4] $\ell \cot\alpha \cot\beta$

Q.20 The angle of elevation of the top of a hill from each of the vertices A, B, C of a horizontal triangle is α. The heigth of the hill is -

[1] b tan α. cosec B [2] $\frac{1}{2}$ a tan α.cosec A

[3] $\frac{1}{2}$c tan α.cosec C [4] both (2) and (3)

ANSWER KEY

Que.	1	2	3	4	5	6	7	8
Ans.	2	4	2	1	3	2	1	2
Que.	9	10	11	12	13	14	15	16
Ans.	2	4	3	3	4	1	2	4
Que.	17	18	19	20				
Ans.	2	1	2	4				

EXERCISE – V NTSE /OLYMPIAD /FOUNDATION PROBLEMS

Q.1 A pole stands vertically inside a triangular park ABC. If the angle of elevation of the top of the pole from each corner of the park is same, then in ΔABC, the foot of the pole is at the **[IIT 2000]**

[1] centroid [2] circumcentre
[3] incentre [4] orthocentre

Q.2 A man from the top of a 100 m high tower sees a car moving towards the tower at an angle of depression of 30°. After some time, the angle of depression becomes 60°. The distance (in metres) travelled by the car during this time is **[IIT 2001]**

[1] $100\sqrt{3}$ m [2] $\frac{200\sqrt{3}}{3}$ m

[3] $\frac{100\sqrt{3}}{3}$ m [4] $200\sqrt{3}$ m

Q.3 The angle of elevation of a stationary cloud from a point 2500 m above a lake is 15° and the angle of depression of its reflection in the lake is 45°. The height of cloud above the lake level is **[IIT, 76]**

[1] $2500\sqrt{3}$ metres [2] 2500 metres

[3] $500\sqrt{3}$ metres [4] none of these

Q.4 PQ is a vertical tower and P , Q are its bottom and top points respectively. A, B, C are three points on a horizontal line passing through P. Angles of elevation of Q from points A,B,C are equal to θ. If a , b, c are sides and Δ is the area of triangle ABC, then the height of the tower is **[IIT, 80]**

[1] $\frac{abc}{4}$ [2] $\frac{abc\tan\theta}{4\Delta}$

[3] $\frac{4abc\,\Delta}{abc}$ [4] $\frac{4\tan\theta}{abc}$

Q.5 The relative positions of four ships A,B,C,D in a sea are as follows : B is on line segment AC ; B is north to D and D is just west to C, BD = 2 km. If $\angle$BDA = 40°. $\angle$BCD = 25°, then distance between A and D is (here sin 25° = 0.423) **[IIT, 83]**

[1] 3.28 km [2] 3.46 km
[3] 4.28 km [4] 4.83 km

Q.6 A ladder is inclined to a wall at angle α. Its foot is drawn upto a distance a and as such it slides a distance b along the wall and makes β angle with the wall. Then a is equal to **[IIT, 85]**

[1] $\frac{b}{2}\tan(\alpha+\beta)$ [2] b tan ($\alpha+\beta$)

[3] $b\tan\frac{\alpha+\beta}{2}$ [4] $\frac{b}{2}\tan\frac{\alpha+\beta}{2}$

Q.7 The angle of elevation of an aeroplane flying horizontally at a height of 1 km is observed to be 60°and after 10 seconds it is observed to be 30°. The uniform velocity of the aeroplane in km/hr is **[IIT, 89]**

[1] $60\sqrt{3}$ [2] 240

[3] $240\sqrt{3}$ [4] None of these

Q.8 ABC is a triangular area where AB = AC = 100 m. A TV tower is standing at the midpoint of BC. If angles of elevation of the top of the tower with respect to A,B,C, are 45°, 60°, 60°, then the height of the tower is **[IIT, 89]**

[1] 50m [2] $50\sqrt{3}$ m

[3] $50/\sqrt{3}$ m [4] None of these

Q.9 A tower AB is inclined to vertical towards towards west at an angle α. The angular elevation of B, the top of the tower is β as observed from a point C due west of A at a distance d from A. If the angular elevation of B from point D due east of C at a distance 2d from C is γ, then 2 tan α can be given as **[IIT, 94]**

[1] 3 cot β – 2 cot γ [2] 3 cot γ – 2 cot β
[3] 3 cot β – cot γ [4] cot β – 3 cot γ

Q.10 The upper 3/4 th portion of a vertical pole subtends an angle $\tan^{-1}$ 3/5 at a point in the horizontal plane through its foot and at a distance 40m from the foot. A possible height of the vertical pole is **[AIEEE, 2003]**

[1] 80 m [2] 20 m
[3] 40 m [4] 60 m

Q.11 A person standing on the bank of a river observes that the angle of elevation of the top of a tree on the opposite bank of the rive is 60° and when he retires 40 meters away from the tree the angle of elevation becomes 30°. The breadth of the river is **[AIEEE 2004]**

[1] 40m [2] 30 m
[3] 20 m [4] 60 m

Q.12 A tower stands at the centre of a circular partk. A and B are two points on the boundary of the park such that AB(=a) subtends an angle of 60º at the foot of the tower, and the angle of elevation of the top of the tower from A or B is 30º. The height of the tower is **[AIEEE 2007]**

[1] $a/\sqrt{3}$ [2] $a\sqrt{3}$
[3] $2a/\sqrt{3}$ [4] $2a\sqrt{3}$

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12
Ans.	2	2	1	2	3	3	3	2	3	3	3	1